11 Complexity Theory and Transport Planning: Fractal Traffic Networks

Erel Avineri

One of the important procedures in many applications of transport planning is the analysis of the performance of traffic networks that is related to the supply and demand of transport (commonly measured by the road capacity and the predicted traffic volume, accordingly). This chapter demonstrates the possibility of incorporating complexity analysis in the modelling of traffic networks. One may argue that many traffic networks are structured in a rather hierarchical form, featuring complex and fractal characteristics of the geometric structure of the networks, as well as the spatial distribution of flows on it. Illustrated by a numeric example we demonstrate the relevance of complexity theory to the analysis of the performance of traffic networks. Conceptual and methodological issues that could be addressed by further research in transport planning are suggested.

11.1 Introduction

Transport systems are comprised of many elements – services, prices, infrastructures, vehicles, control systems and users, which are been taken by transport planners as a whole. Characteristics and performances of transport systems and their components are usually defined on the basis of quantitative evaluation of their effects, considering the objectives and the constraints of the transport system. The planning, design and operation of transport systems may vary much by its focus and methodologies. For example, a detailed level of representation may be used in the functional design and control management of transport systems, while in the transport planning of a region only the major roads of the traffic network may be represented. One of the important aspects of the transport planning process is assigning the predicted traffic volumes on the road segments. This is followed by an analysis of the transport network and its performance, mainly by observing its ability to satisfy the travel demand of persons and goods in a given area. The scale of the network is generally depended on the stage of the planning and its detailed level.

Segments of the traffic network (such as road links) can be classified into hierarchies according to the size and type of traffic flow they are supposed to carry. Public roads may be classified by types, such as motorways, trunk roads, principle roads and local B and C type

1 Reader in Travel Behaviour at the Centre for Transport & Society, Faculty of Environment and Technology, University of the West of England, Bristol, United Kingdom.
roads. The design of such roads is based on their purposes and their roles in accessibility and mobility. This leads to specific recommendations on the level of service provided by different types of road, and guidelines on the geometric design of such roads (such as the number of lanes and the lanes width, and the design of other elements such as intersections, parking, and traffic calming). Table 11.1 represents the length of public roads in Great Britain. While Motorways and trunk roads carry large volumes of transport, they hardly represent the size of the transport system (motorways themselves carry about a quarter of the traffic volume although they account for only 1% of the total length of the transport network). This illustrates a dilemma transport planners are faced with; in order to reduce the complexity of the planning process, would it be enough to analyse a network comprises only of main roads, but representing most of the traffic volumes?

<table>
<thead>
<tr>
<th>Road Type</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorways</td>
<td>3,466</td>
</tr>
<tr>
<td>Trunk</td>
<td>12,189</td>
</tr>
<tr>
<td>Principal</td>
<td>42,446</td>
</tr>
<tr>
<td>B roads</td>
<td>30,189</td>
</tr>
<tr>
<td>C roads</td>
<td>84,459</td>
</tr>
<tr>
<td>Unclassified roads</td>
<td>223,184</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>388,008</strong></td>
</tr>
</tbody>
</table>

Table 11.1 Public Roads in Great Britain - Length by Class of Roads (based on UK National Statistics, DfT, 2005).

It has been observed that transport networks may exhibit properties of regularity and self-similarity (Thomson, 1977, Benguigui & Daoud, 1991, Marshall, 2005). Complexity properties of the traffic network have been applied in different contexts. Marshall (2005) uses complexity properties in order to gather a better understanding of the relationships between streets and patterns, forming a basis for a broader framework that may be used to underpin streets-oriented urban design agendas.

The conventional process of transport modelling involves generating and analysing a traffic network, which is a sub-network of the physical road network. The represented network may include existing components, planned ones, or a mix. Physical dynamic changes in the road network often consist of adding new capacities or deleting existing capacities. While some of these changes may already been introduced to the transport system, others may be suggested and considered as part of an alternative plan to be appraised. Transport modellers do not have access to detailed information about the minor roads, and are not able to calibrate the travel time functions on these segments. Moreover, when modelling a planned transport system, some of its elements may not be fully planned or designed. For example, the planning of some minor roads in residential areas may be based on partial information on its geometric shape and capacity. Another unknown in some cases may be the expected traffic flow measures as a function of the traffic volumes. For example, the link volume-travel time functions may be described as following:
\[ t_{ij} = \alpha_{ij} + \beta_{ij} f_{ij} \]  

where 

- \( t_{ij} \) is the travel time on link \( ij \) 
- \( \alpha_{ij} \) is the free flow time on link \( ij \) 
- \( \beta_{ij} \) is the delay parameter for link \( ij \) (the increase in travel time per unit increase in the flow on link \( ij \)), and 
- \( f_{ij} \) is the flow on link \( ij \).

The above notation is based on the assumption of linear relationship between traffic volume on a link and the travel time on it.

The addition of a link to an existing transport system may lead to undesired situations. For example, in heavily congested areas, some of the medium and long-distance trips use minor roads as ‘rat runs’ (see p. 365 in Ortúzar & Willumsen, 2001).

In a town planning context, some researchers argue that the complex characteristics of urban traffic networks can be explained by a fractal growth of cities (or vice versa) (Shen, 1997; Chen & Lou, 1998; Lu & Tang, 2004; Yerra & Levinson, 2005). Most of these studies explore the complexity of traffic network in the context of its spatial characteristics.

### 11.2 Definitions

Marshall (2005) introduced several concepts to define the complex structure of traffic networks. \textit{Depth} is a measure of network ‘distance’ – steps of adjacency – between network components. Calculating the depth of any axial line to any other, the average depth of a network can be measured. \textit{Recursivity} is defined by the number of depths divided by the number of routes (where the number of depths is simply equivalent to the maximum depth). \textit{Complexity} is defined as the number of distinct types of route present over and above the number of distinct types generated by difference in depth alone (which is the number of distinct types present less the value of the maximum depth) divided by the number of routes. (for illustrative examples see Marshall, 2005). Many traffic networks can be described as fractals. While some mathematicians avoid giving the strict definition, \textit{fractal} is a geometric object that is generally "a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole" (Mandelbrot, 1982).

Traffic flow parameters of link \( ij \) (\( \alpha_{ij} \), \( \beta_{ij} \)) may be typical to distinct route type or depth. Different types of road links, classified by hierarchical configuration of the traffic network, may be assigned with a limited range of parameter values to represent the typical traffic flow characteristics on such road types. A specific issue this chapter is dealing with is the effect of introducing a new link to a congested traffic network, and the likelihood of this additional capacity to improve the system’s performance (traditionally measured by users’ aggregated cost or travel time) The ratios between traffic flow parameters of two roads are assumed to be related to the difference in the hierarchical level of these roads, as follows:
\[ \alpha^n = k^{n-1} \alpha; \quad \beta^n = l^{n-1} \beta \]  

(2)

where \( \alpha \) and \( \beta \) are traffic flow parameters typical to roads classified by the highest hierarchical level of the traffic network; assuming recursivity and self-similarity of the traffic flow characteristics in different hierarchical levels, \( \alpha^n \) and \( \beta^n \) are used to represent traffic flow parameters typical to roads classified by the \( n \)-hierarchical level of the traffic network. Assumptions about fixed linear ratios are made for demonstration purposes only, and may be over simplistic to represent the fractal nature of road hierarchy in real traffic networks.

Following this recursive definition of traffic flow parameters (Eq. 2), traffic networks (or sub-networks) that have a recursivity of about \( l \) and a complexity of about \( 0 \) (both properties as defined by Marshall, 2005) may be described as fractals. The recursivity of traffic flow measures can be represented by \( k \) and \( l \) (see Eq. 2). Complexity properties of roads’ traffic flow parameters may help transport planning to determine under what circumstances the addition of a new link to an existing transport network improves or worsens the overall system performance, as demonstrated in the next section.

11.3 Numeric Example

Let us consider a simple network problem, represented in Figure 11.1. The nodes of the traffic network are represented by a, b, c, and d. There are three possible paths to get from the origin to the destination, and the total traffic, \( Q \), is equal to the sum of traffic volumes on these paths (\( F_1, F_2, F_3 \)):

\[ Q = F_1 + F_2 + F_3 \]  

(3)

where:

\[ f_{ab} = F_1 + F_3; f_{bd} = F_1; f_{ac} = F_2; f_{cd} = F_2 + F_3; f_{bc} = F_3 \]  

(4)

And the travel time experienced by travellers on each of the paths are:

\[ T_1 = T_{ab} + T_{bd}; T_2 = T_{ac} + T_{cd}; T_3 = T_{av} + T_{bc} + T_{cd} \]  

(5)

As in the classical problem for Braess’ paradox, we assume that the problem is symmetric, so that:

\[ \alpha_{ab} = \alpha_{cd} = 0 \]
\[ \alpha_{bd} = \alpha_{bd} = \alpha' \]
\[ \beta_{ab} = \beta_{cd} = \beta' \]
\[ \beta_{bd} = \beta_{bd} = 0 \]

The parameters were used in this numeric example are presented in Table 11.2. These parameter values were used in Braess’ (1968) classical example.
This is a pre-publication version of the following article:

Figure 11.1  The Traffic Network and three possible paths.

<table>
<thead>
<tr>
<th>link (ij)</th>
<th>( g_{ij} )</th>
<th>( \beta_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ab</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>bd</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>ac</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>cd</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>bc</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11.2  Parameter values of the road segments.

Link bc has a lower hierarchical level in the traffic network. Following Eq. 2, its traffic flow parameters may be represented as follows:

\[
\alpha_{bc} = \alpha^2 = k \alpha; \quad \beta_{bc} = \beta^2 = l \beta
\]  

(6)

Network equilibrium models are commonly used for the prediction of traffic flow on the network links. Wardrop (1952) provided a simple behavioural principle to describe the distribution of traffic volume on a traffic network, widely used in transport planning. Each user seeks to minimize travel cost or time. The traffic flows that satisfy this principle are usually referred to as "user equilibrium", since each user chooses the route that is the best for him. In equilibrium, no user can decrease his/her route travel time by unilaterally switching routes. This condition may be represented by

\[
T_1 = T_2 = T_3
\]  

(7)

The user equilibrium solution for this 3-path network with overall traffic volume of \( Q=6 \) is \( T_1=T_2=T_3=88.3 \) minutes, while the user equilibrium solution for a 2-path network (by omitting the link bc) is \( T_1=T_2=80 \) minutes. This illustrates the Braess’ Paradox (Braess, 1968): adding new capacity (such as an extra link) in a congested network does not necessarily reduce congestion and can even increase it. This situation happens because the users of the network do not face the true social cost of an action.
Pas & Principuo (1997) do not find it appropriate to refer to this phenomenon as a paradox. Exploring the general expression of the network presented in figure 11.1, they found that Braess’ paradox occurs when:

\[
\frac{2(\alpha^1 - \alpha^2)}{3\beta^1 + \beta^2} < Q < \frac{2(\alpha^1 - \alpha^2)}{\beta^1 - \beta^2}
\] (8)

For \( Q \) values outside of the above region, Braess’ paradox does not occur, either because the demand is too low or because the demand is too high. Pas & Principuo (1997) show that for the values of Braess’ original example, the paradox occurs for \( Q \) value between 2.58 and 8.89. Can the occurrence of Braess’ paradox in a traffic network be predicted by the complexity properties introduced in the previous section? Using the recursive definitions of traffic flow parameters (Eq. 2), we find that Braess paradox occurs when:

\[
\frac{2\alpha(1-k)}{\beta(3+l)} < Q < \frac{2\alpha(1-k)}{\beta(1-l)}
\] (9)

The depth represented in this equation is 2, but the hierarchical difference between the added link and the existing routes may be more than that. Eq. 9 can be generalised for an added link of \( n \)-hierarchical level, Eq. 9 as follows:

\[
\frac{2\alpha(1-k^n)}{\beta(3+l^n)} < Q < \frac{2\alpha(1-k^n)}{\beta(1-l^n)}
\] (10)

It is easy to see that Braess’ paradox does not occur when \( l^{n-1} > 1 \) and \( k^{n-1} < 1 \), or when \( l^{n-1} < 1 \) and \( k^{n-1} > 1 \). The region of \( Q \)-values that satisfies Eq. 10 depends on the depth of the sub-network \( n \) and the recursivity of the hierarchical traffic flow properties. Thus, exploring the sensitivity of Eq. 10 to these values, the likelihood of Braess’ paradox effects may be predicted for given estimation of traffic flow entering the sub-network \( Q \). The probability of overall performance of the network to be worse off than before a link is added \( p(T_{new} > T_{old}) \) (or in other words, the probability of Braess’ paradox to occur) is:

\[
p(T_{new} > T_{old}) = \sum_q p(q) \cdot \pi(q,k,l,n) \cdot J(q,k,l,n)
\] (11)

Travel demand has a stochastic nature, thus the traffic volume that enter a traffic network is changing over time, reflecting changes in the demand for travel. In Eq. 11 \( p(q) \) stands for the probability that the traffic volume enters the network is \( q \).
The following example illustrates how the complex nature of the traffic network may be related to the probability of Braess’ paradox to occur.

The distribution of the traffic volume entering the network presented in Figure 11.1 is estimated as follows: \( Q \) (traffic flow) is 0 with a probability of 0.05, 1 with a probability of 0.1, 2 with a probability of 0.2, 3 with a probability of 0.3, 4 with a probability of 0.2, 5 with a probability of 0.1, 6 with a probability of 0.05. The traffic flow parameters are presented in Table 11.2.

The probability \( p(T_{\text{new}} > T_{\text{old}}) \) for different levels of hierarchy \((0 \leq k \leq 2 \text{ and } 0 \leq l \leq 2)\) and for a depth of 2 is represented in Figure 11.2. The probability \( p(T_{\text{new}} > T_{\text{old}}) \) for different measures of hierarchy \((0 \leq k \leq 2 \text{ and } 0 \leq l \leq 2)\) and for a depth of 4 is represented in Figure 11.3. A general property of the traffic network’s complexity may be observed here: the higher the depth of the network system is, the lower is the probability of Braess’ paradox to occur, and the safer is the assumption that introducing a new link will improve the overall performance of the system. A transport modeller may find use in such analysis even if he is not provided with accurate and precise properties of the traffic complexity.

11.4 Conclusion

A traffic network can be described as a complex system; one may argue that it may be characterised by self-similarity and symmetry across scales. This chapter describes conceptual and methodological issues for presenting and exploring complex traffic systems and illustrates the application of complexity properties in the analysis of traffic networks.

Facing rather large and complex traffic network, parts of it may not be completely planned or design, transport modellers may use complexity analysis of the traffic network in order to identify situations where added capacity might not be followed by improved performance of the traffic system. Using complexity terms presented in this work, as well as other complexity terms, in the description of traffic networks can form the structural basis for a broader modelling framework.

Complexity modelling of the traffic network may also help in the conceptual representation of the minor parts of the road network, which in many cases are not represented fully due to limited knowledge of the modeller or limited resources by him (see, for example, Bovy & Hansen, 1983). The level of spatial detail (i.e. zone side and network detail) used in transport analysis is an important factor affecting the correctness and the
accuracy of the resulting estimates of the impact of changes in the transport system.

In order to incorporate complexity properties into traffic modelling, further empirical and theoretical investigation of the complexity of road networks in the context of traffic flow are required. The example presented in this chapter is rather simple and does not address other dimensions of real traffic networks, such as recursivity or connectivity. An obvious generalization of the presented approach is to develop a systematic analysis of any complex traffic network.

Figure 11.2 Probability of Braess’ Paradox as a function of the Network Recursivity $0 \leq k \leq 2, 0 \leq l \leq 2, n=2$

Figure 11.3 Probability of Braess’ Paradox as a function of the Network Recursivity $0 \leq k \leq 2, 0 \leq l \leq 2, n=4$
References