EXPERIMENTAL AND ANALYTICAL INVESTIGATIONS OF
BRICK MASONRY UNDER COMPRESSIVE FATIGUE LOADING

SPYRIDOULA-IRIS KOLTSIDA

A thesis submitted in partial fulfilment of the requirements of the University of the West of England, Bristol for the degree of Doctor of Philosophy

Faculty of Environment and Technology, University of the West of England, Bristol

October 2017
Do not try to understand everything, lest you become ignorant of everything.

Democritus, 400 BC
# CONTENTS

CONTENTS .......................................................................................................................... i

LIST OF FIGURES ............................................................................................................... iv

LIST OF TABLES .................................................................................................................. xiii

ABSTRACT ............................................................................................................................. xv

ACKNOWLEDGMENTS ......................................................................................................... xvi

GLOSSARY OF TERMS ......................................................................................................... xviii

1. INTRODUCTION ............................................................................................................. 1

   1.1 Background ................................................................................................................. 1

   1.2 Research aim and objectives ....................................................................................... 9

   1.3 Structure of the thesis ................................................................................................. 10

2. REVIEW OF LITERATURE ............................................................................................. 13

   2.1 Introduction .................................................................................................................. 13

   2.2 Overview of fatigue ..................................................................................................... 13

       2.2.1 Definitions ........................................................................................................... 13

       2.2.2 Historical background to fatigue ....................................................................... 16

       2.2.3 Fatigue of materials as a physical phenomenon ............................................ 20

   2.3 Fatigue in brittle materials ......................................................................................... 21

       2.3.1 Behaviour of concrete under long-term fatigue loading .................................. 22

   2.4 Behaviour of masonry under quasi-static and long-term fatigue compressive

       loading ........................................................................................................................... 27

       2.4.1 Masonry under quasi-static compressive loading ........................................... 28

       2.4.2 Masonry under long-term fatigue loading ..................................................... 31

   2.5 Chapter summary ....................................................................................................... 46

3. RESEARCH DESIGN AND METHODOLOGY .................................................................. 47

   3.1 Introduction .................................................................................................................. 47

   3.2 Outline of the research ............................................................................................... 47

   3.3 Experimental Design .................................................................................................. 49
### 3.3.1 Materials ................................................................. 49
### 3.3.2 Test specimens .......................................................... 52
### 3.3.3 Experimental tests ....................................................... 54
### 3.4 Experimental Techniques .................................................. 54
### 3.4.1 Instrumentation .......................................................... 55
### 3.4.2 Experimental testing of bricks ......................................... 57
### 3.4.3 Experimental testing of mortar ........................................ 59
### 3.4.4 Experimental testing of masonry prisms ............................. 60
### 3.5 Data analysis .................................................................. 67
### 3.6 Chapter summary ............................................................ 68

### 4. EXPERIMENTAL RESULTS ................................................. 69

#### 4.1 Introduction ................................................................ 69
#### 4.2 Results of experimental tests on bricks and mortar .......... 69
    #### 4.2.1 Quasi-static compressive tests on bricks .................. 70
    #### 4.2.2 Quasi-static compressive tests on mortar ................. 71
#### 4.3 Results of experimental tests on masonry prisms .............. 73
    #### 4.3.1 Quasi-static compressive tests on masonry prisms ....... 73
    #### 4.3.2 Long-term compressive fatigue tests on masonry prisms 76
    #### 4.3.3 Pilot long-term fatigue tests on masonry prisms for studying change in stiffness .................................................. 92
#### 4.4 Discussion ................................................................... 110
#### 4.5 Chapter summary .......................................................... 113

### 5. ANALYSIS ................................................................... 114

#### 5.1 Introduction ................................................................ 114
#### 5.2 S-N curves for masonry ................................................... 114
    #### 5.2.1 Available S-N models ........................................... 115
    #### 5.2.2 Proposed S-N-P model ............................................ 120
#### 5.3 Changes in strain (ε-N) during fatigue deterioration ........ 129
    #### 5.3.1 Available ε-N models ............................................. 130
    #### 5.3.2 Proposed ε-N model .............................................. 133
    #### 5.3.3 Proposed model for elastic and plastic strain evolution .. 144
5.4 Changes in the Young’s modulus (E) during fatigue deterioration .......... 160
5.4.1 Available models for the Young’s modulus ........................................ 160
5.4.2 Proposed model for the Young’s modulus ........................................ 160
5.5 Practical Example .................................................................................. 171
5.6 Discussion .............................................................................................. 179
5.6.1 S-N curves for masonry ...................................................................... 179
5.6.2 Changes in strain (ε-N) during fatigue deterioration ......................... 186
5.6.3 Changes in the Young’s modulus (E) during fatigue deterioration .... 188
5.7 Chapter summary .................................................................................. 188
6. CONCLUSIONS AND RECOMMENDATIONS ........................................ 190
6.1 Introduction .......................................................................................... 190
6.2 Review of research objectives ............................................................... 190
6.3 Research limitations ............................................................................ 194
6.4 Review of research findings ................................................................... 196
6.5 Application of the research findings ..................................................... 200
6.6 Contribution to the body of knowledge ............................................... 201
6.7 Conclusions .......................................................................................... 203
6.8 Recommendations for future research ................................................. 205
REFERENCES ............................................................................................ 207
LIST OF FIGURES

Figure 1-1 Distribution of bridge by type and ownership in the UK. ........................................1
Figure 1-2 Little Stanney bridge No 141 over the Shropshire Union canal (Biggs, 2012) .....2
Figure 1-3 Waterways of England and Wales (Canal & River Trust, 2011). .......................3
Figure 1-4 Changes in locomotive (a) weight and (b) axle load with time for the UK (Hayward, 2011)...........................................................................................................5
Figure 1-5 Licensed vehicles by type since 1909 in semi-logarithmic scale created from data (Department for Transport, 2016).....................................................................................6
Figure 1-6 Vehicle miles by vehicle type in Great Britain since 1949 (Department for Transport, 2016) ........................................................................................................................................6
Figure 1-7 Variation of heavy vehicle load with time (Ryall, 2008;). ....................................7
Figure 1-8 Outline of the thesis structure. ..................................................................................10
Figure 2-1 Configuration of cyclic loading .............................................................................14
Figure 2-2 Form of (a) S-N curve, (b) Goodman diagram .....................................................16
Figure 2-3 Different periods constituting the fatigue life of materials. ..................................20
Figure 2-4 Effect of cyclic loading on the stress-strain curve (Holmen, 1982) ....................25
Figure 2-5 Typical Strain-Time curve for concrete under fatigue loading (Medeiros et al., 2015).............................................................................................................................26
Figure 2-6 Typical Stiffness-Number of loading cycles curve for concrete (Alliche, 2004). 27
Figure 2-7 Triaxial state of stress at the interface of brick and mortar of masonry ...........29
Figure 2-8 Typical stress-strain curve for masonry in compression, 1) typical, 2) idealised diagram, 3) design diagram (Eurocode 6, 2012)......................................................................................30
Figure 2-9 Envelope, stability point and the common point curves for brick masonry loaded normal and parallel to the bed joints (Alshebani, 2013). ........................................34
Figure 2-10 Three types of masonry prisms used by Roberts et al. (2006) ........................................... 36
Figure 2-11 Test data of brickwork prisms under very high sustained compressive cyclic loading by Ronca et al. (2004) .......................................................................................... 37
Figure 2-12 Failure mechanisms under fatigue loading (Ronca et al., 2004) ................................. 38
Figure 2-13 Ring separation collapse mechanism under cyclic loading (Melbourne et al., 2007). ........................................................................................................................................................................... 39
Figure 2-14 SMART assessment procedure ............................................................................................ 40
Figure 2-15 Joint failure model indicating stress vs. time or cycles to failure (ST or SN curves) for static, fatigue and creep loading (Tomor & Verstrynge, 2013). ................................. 43
Figure 2-16 Typical ε-N curve for masonry under fatigue loading (Carpinteri et al., 2014). .......... 45
Figure 3-1 Flow chart of research activities and links between different research methods used in this work. ........................................................................................................................................ 48
Figure 3-2 B1M1 masonry prism in accordance with ASTM standards used for the experimental tests (dimensions in mm). .................................................................................... 52
Figure 3-3 Masonry prism in accordance with the European standards for testing the compressive strength of masonry. ........................................................................................................ 53
Figure 3-4 250 kN servo-controlled hydraulic actuator used for quasi-static and fatigue testing of masonry. ........................................................................................................................................ 56
Figure 3-5 Setup of tests on bricks under quasi-static compressive loading. ................................. 58
Figure 3-6 Test setup and instrumentation used for testing masonry prisms under quasi-static and cyclic compressive load ........................................................................................................ 62
Figure 3-7 Theoretical Wöhler or Stress –Number of loading cycles (SN) curve ..................... 63
Figure 3-8 Sinusoidal configuration of load used for fatigue testing of masonry prisms. .. 64
Figure 3-9 Loading sequence of three distinct branches A) quasi-static, B) cyclic, C) unloading ................................................................. 66

Figure 4-1 Typical stress–strain curve for B1 bricks under quasi-static compression........ 71
Figure 4-2 Typical stress-strain curve for M1 hardened mortar under quasi-static compression............................................................................. 72
Figure 4-3 Typical stress-strain curve for B1M1 prisms under quasi-static compression... 74
Figure 4-4 Crack pattern and spalling of a masonry prism at ultimate failure under quasi-static compression [a] front view and [b] narrow side of the prism......................... 75
Figure 4-5 Typical stress-strain curves for B1 brick, M1 mortar and B1M1 masonry prism ...................................................................................... 76
Figure 4-6 Experimental data of long-term fatigue tests under compression for B1M1 type masonry (n=38)........................................................................................... 79
Figure 4-7 Total longitudinal strain variation with the cycle ratio for 55% maximum stress level (a) maximum total strain, (b) minimum total strain ........................................ 80
Figure 4-8 Total longitudinal strain variation with the cycle ratio for 60% maximum stress level (a) maximum total strain, (b) minimum total strain ........................................ 81
Figure 4-9 Total longitudinal strain variation with the cycle ratio for 68% maximum stress level (a) maximum total strain, (b) minimum total strain ........................................ 82
Figure 4-10 Total longitudinal strain variation with the cycle ratio for 80% maximum stress level (a) maximum total strain, (b) minimum total strain ........................................ 83
Figure 4-11 Strain rate of stage II with the maximum stress level (n=30) ....................... 84
Figure 4-12 Strain rate of stage II with loading cycles to failure for 55%, 60%, 68% and 80% maximum stress (n=30) .................................................................................. 85
Figure 4-13 Minimum, maximum and mean recorded strains at the end of stage I for different maximum stress levels (n=30) ................................................................. 87
Figure 4-14 Minimum, maximum and mean recorded strains at the end of stage II for different maximum stress levels (n=30) ................................................................. 87
Figure 4-15 Minimum, maximum and mean recorded strains at the end of stage III for different maximum stress levels (n=30) ................................................................. 88
Figure 4-16 Evolution of strain with the number of cycles to failure at the end of stage I (n=30) ............................................................................................................................... 89
Figure 4-17 Evolution of strain with the number of cycles to failure at the end of stage II (n=30) ............................................................................................................................... 89
Figure 4-18 Evolution of strain with the number of cycles to failure at the end of stage III (n=30) ............................................................................................................................... 90
Figure 4-19 Representative failure modes of masonry prisms under fatigue compression loading at 55%, 60%, 68% and 80% maximum stress level ................................................. 91
Figure 4-20 Experimental data of long-term fatigue tests under compression for B1M1 type masonry enriched with data for 63%, 68% and 73% stress levels (n=70) ................. 94
Figure 4-21 Coefficient of Variation of fatigue test data with the maximum stress level ................................ ................................ ................................ ................................ 95
Figure 4-22 [a] Stress-strain curve and [b] total longitudinal strain variation for B1M1- 58 at 68% maximum stress with number of cycles ........................................................................... 97
Figure 4-23 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 59 at 68% maximum stress with number of cycles ........................................................................... 98
Figure 4-24 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 61 at 68% maximum stress with number of cycles ........................................................................... 99
Figure 4-25 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 62 at 68% maximum stress with number of cycles ................................................................. 100
Figure 4-26 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 63 at 68% maximum stress with number of cycles ................................................................. 101
Figure 4-27 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 64 at 68% maximum stress with number of cycles ................................................................. 102
Figure 4-28 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 74 at 63% maximum stress with number of cycles ................................................................. 103
Figure 4-29 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 80 at 63% maximum stress with number of cycles ................................................................. 104
Figure 4-30 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 82 at 63% maximum stress with number of cycles ................................................................. 105
Figure 4-31 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 83 at 63% maximum stress with number of cycles ................................................................. 106
Figure 4-32 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 86 at 63% maximum stress with number of cycles ................................................................. 107
Figure 4-33 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 88 at 63% maximum stress with number of cycles ................................................................. 108
Figure 4-34 Strain at 63% and 68% stress levels together with strain during quasi-static tests (n=18) .................................................................................................................................. 109
Figure 5-1 Fatigue test data together with the fatigue model proposed by Roberts et al. (2006) ..................................................................................................................................... 116
Figure 5-2 Fatigue test data together with the fatigue model proposed by Casas (2009) for probability of survival 95% and 50%........................................................................................................ 118
Figure 5-3 Fatigue test data together with the fatigue model proposed by Tomor and Verstrynge (2013).................................................................................................................................................................................. 120

Figure 5-4 Variation of failure probability with the loading cycles for different stress levels. ........................................................................................................................................................................................................................................... 123

Figure 5-5 Experimental data and predicted S-N curves for different probabilities of failure. ........................................................................................................................................................................................................................................................................ 124

Figure 5-6 S-N-P curves for masonry under fatigue loading at 2Hz with 10% minimum stress........................................................................................................................................................................................................................................................................ 129

Figure 5-7 Curve fitting for strain test data against maximum stress for the intersection point between stage I and stage II of the fatigue life........................................................................................................................................................................................................................................................................ 136

Figure 5-8 Curve fitting for the strain rate test data against maximum stress for stage II of the fatigue life........................................................................................................................................................................................................................................................................ 136

Figure 5-9 Curve fitting for strain test data at failure against maximum stress ........................................................................................................................................................................................................................................................................ 140

Figure 5-10 Total strain and proposed mathematical model against number of cycles ratio for (a) 55%, (b) 60% ........................................................................................................................................................................................................................................................................ 142

Figure 5-11 Total strain and proposed mathematical model against number of cycles ratio for (a) 68%, (b) 80% ........................................................................................................................................................................................................................................................................ 143

Figure 5-12 Schematic of stress - strain curve for quasi-static loading and elastic and plastic components of strain ........................................................................................................................................................................................................................................................................ 144

Figure 5-13 Changes in total strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress ........................................................................................................................................................................................................................................................................ 146

Figure 5-14 Changes in plastic strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress ........................................................................................................................................................................................................................................................................ 147
Figure 5-15 Changes in elastic strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress ................................................................. 148

Figure 5-16 Strain against number of cycles ratio for (a) \( \varepsilon_{pl}/\varepsilon_{tot} \) and (b) \( \varepsilon_{el}/\varepsilon_{tot} \) for 63% maximum stress .................................................................................. 149

Figure 5-17 Strain against number of cycles ratio for (a) \( \varepsilon_{pl}/\varepsilon_{tot} \) and (b) \( \varepsilon_{el}/\varepsilon_{tot} \) for 68% maximum stress .................................................................................. 150

Figure 5-18 Curve fitting for plastic strain test data against maximum stress for the intersection point between stage I and stage II of the fatigue life ....................... 152

Figure 5-19 Curve fitting for the plastic strain rate test data against maximum stress for stage II of the fatigue life ............................................................................ 153

Figure 5-20 Curve fitting for plastic strain test data against maximum stress at failure .. 156

Figure 5-21 Test data and proposed mathematical model against number of cycles ratio for (a) plastic (\( \varepsilon_{pl}/\varepsilon_{tot} \)) and (b) elastic (\( \varepsilon_{el}/\varepsilon_{tot} \)) strain for 63% maximum stress ................ 158

Figure 5-22 Test data and proposed mathematical model against number of cycles ratio for (a) plastic (\( \varepsilon_{pl}/\varepsilon_{tot} \)) and (b) elastic (\( \varepsilon_{el}/\varepsilon_{tot} \)) strain for 68% maximum stress ................ 159

Figure 5-23 Schematic representation of the way Young’s modulus was calculated as a secant modulus between the minimum and maximum stress ......................... 161

Figure 5-24 Changes in Young’s modulus against number of cycles ratio for (a) 55% and (b) 60% maximum stress ................................................................................. 162

Figure 5-25 Changes in Young’s modulus against number of cycles ratio for (a) 68% and (b) 80% maximum stress ................................................................................. 163

Figure 5-26 Changes in Young’s modulus against number of cycles ratio for (a) 63% and (b) 68% maximum stress ................................................................................. 164

Figure 5-27 Residual over initial Young’s modulus against the maximum stress ......... 165
Figure 5-28 Slope defined by the initial and the residual Young’s Modulus at failure against the maximum stress (n=42)............................................................................................................ 166

Figure 5-29 Changes in Young’s modulus against number of cycles ratio together with prediction model for (a) 55% and (b) 60% maximum stress ................................................................. 168

Figure 5-30 Changes in Young’s modulus against number of cycles ratio together with prediction model for (a) 68% and (b) 80% maximum stress ................................................................. 169

Figure 5-31 Changes in Young’s modulus against number of cycles ratio together with prediction model for (a) 63% and (b) 68% maximum stress ................................................................. 170

Figure 5-32 Layout of the Cavone Bridge ......................................................................................... 172

Figure 5-33 Representation of the axle positions for fatigue Model 3 according to Eurocode 1 (2002)........................................................................................................................................ 173

Figure 5-34 SN curve for 0.95 probability of survival, maximum stress level corresponding to main arches and number of sustained cycles to date for (a) KL1 and (b) KL3. ............ 176

Figure 5-35 SN curve for 0.99 probability of survival, maximum stress level corresponding to secondary arch arches and number of sustained cycles to date for (a) KL1 and (b) KL2. ........................................................................................................................................... 177

Figure 5-36 Strain-loading cycles curve for 79% maximum stress .................................................. 178

Figure 5-37 Young’s modulus-loading cycles curve for 79% maximum stress ................................. 179

Figure 5-38 Test data by Clark (1994) with proposed S-N-P curves (n=15)................................. 180

Figure 5-39 Dry test data by Roberts et al. (2006) with proposed S-N-P curves for different minimum stress levels (n=13) .................................................................................................................. 182

Figure 5-40 Test data by Tomor et al. (2013) with proposed S-N-P curves (n=13)................. 183

Figure 5-41 Comparison of models proposed as lower limits for the fatigue life of masonry (n=64) ........................................................................................................................................... 185
Figure 5-42 Comparison of models proposed to predict the mean fatigue life of masonry (n=64)..................................................................................................................................................186
LIST OF TABLES

Table 2-1 Relationships between basic characteristics of cyclic loading ......................... 15
Table 3-1 Brick mechanical properties and dimensions (product specifications).............. 50
Table 3-2 Fresh mortar mix (BS EN 998-2:2010) .............................................................. 51
Table 3-3 Different masonry types identified to represent different masonry arch bridges (Tomor & Wang, 2010) ........................................................................................................ 51
Table 3-4 Description of the three loading branches for the final test design ................. 66
Table 4-1 Results of quasi-static compression tests on B1 bricks (n=6) ......................... 70
Table 4-2 Results of quasi-static compression tests on hardened M1 mortar cubes ....... 72
Table 4-3 Results of quasi-static compression tests on B1M1 prisms ............................... 74
Table 4-4 Long-term fatigue data for 55% of the maximum compressive stress ............ 77
Table 4-5 Long-term fatigue data for 60% of the maximum compressive stress ............ 77
Table 4-6 Long-term fatigue data for 68% of the maximum compressive stress ............ 77
Table 4-7 Long-term fatigue data for 80% of the maximum compressive stress ............ 78
Table 4-8 Duration of different stages of fatigue for each prism .................................... 86
Table 4-9 Fatigue test results for 73% maximum stress level ......................................... 92
Table 4-10 Fatigue test results for 63% maximum stress level ....................................... 93
Table 4-11 Fatigue test results for 68% maximum stress level ....................................... 94
Table 5-1 Parameters of the fatigue equation proposed by Casas (2009) depending on the required confidence level ................................................................. 117
Table 5-2 Number of cycles to failure and probability of failure at 55% and 60% maximum stress levels ........................................................................................................ 121
Table 5-3 Number of cycles to failure and probability of failure at 63% and 68% maximum stress levels ........................................................................................................ 121
Table 5-4 Number of cycles to failure and probability of failure at 73% and 80% maximum stress levels...

Table 5-5 Mean and standard deviation of logN for different fatigue stress levels...

Table 5-6 Parameters A and B for different probabilities of failure...

Table 5-7 Number of heavy vehicles expected per year and per slow lane according to Eurocode 1 (2002)...

Table 5-8 Knowledge levels according to Italian Code (NTC-08, 2009)...

Table 5-9 Stresses in the main and secondary arches according to Laterza et al., 2016...
ABSTRACT

About 40% of bridges in European transport network are masonry arch bridges (most built over 100 years ago) and are being subjected to increasing loading regimes. Assessment of the long-term fatigue capacity of masonry bridges is necessary to ensure that increased traffic loading does not result in premature deterioration and/or reduce their life expectancy. The study investigates the influence of compressive fatigue loading on the behaviour and mechanical properties of low-strength brick masonry, relevant to the structural loadbearing elements, for example the arch ring in canal masonry bridges. Masonry prisms were tested (n=70) under quasi-static and long-term fatigue loading to collect information on the number of loading cycles under a range of stress levels, changes in the stress-strain curves, evolution of strain and Young’s modulus during fatigue deterioration. Laboratory tests were performed under maximum stress levels between 55–80% of the compressive strength, at 2Hz frequency for a maximum of $10^7$ loading cycles. Test data were analysed to develop analytical expressions to predict the response of masonry under fatigue loading. Test results reveal that fatigue deterioration is characterised by three distinct stages in strain evolution and stress-strain curves. The Young’s modulus decreased by 25%, while the maximum recorded strain increased up to 5.25 times. An expression for the stress - number of cycles - probability (S-N-P) curves was proposed based on probabilistic analysis to predict the fatigue life of masonry at any desired probability. A set of three formulas were developed to predict strain evolution at different stages of fatigue life and a linear equation was derived for the evolution of the Young’s modulus. The proposed S-N-P model can provide numerical data for fatigue analysis of low-strength masonry arch bridges, e.g. for the SMART method to evaluate the remaining service under any traffic loading level. The rate of change in strain can provide useful reference data for long-term monitoring to identify the stage of the fatigue life the structure is experiencing. A reduction factor for the Young’s modulus between 0.9–0.75, depending on the stress level, can be used for assessing masonry arch bridges under fatigue loading. As a consequence, it is recommended that the models describing changes of the mechanical properties of masonry with loading cycles can be adapted by finite element software packages to develop time-dependant models for the analysis of masonry under fatigue.
ACKNOWLEDGMENTS

Writing a doctoral thesis is a long individual journey, which, can not be realised without the help, support and advice of a number of individuals.

First and foremost, I would like to express my gratitude to my supervisors Dr Colin Booth and Dr Adrienn Tomor for their advice, guidance and encouragement during the three years of this research. Their contribution was crucial and enabled me to understand the needs of research, to plan my work and to achieve the objectives in an environment of collaboration and friendly debate.

I gratefully acknowledge the bursary received towards my PhD from the University of the West of England and the International Union of Railways (UIC), which covered my fees and living expenses during my studies. Not having to worry about the cost of living allowed me to focus on the needs of my research and complete the thesis within the expected time limits.

I would also like to thank my colleagues, Louise King, Abdul-Majeed Mahamadu and Thep Lam Thanh for the long conversations. Through sharing their thoughts and having the same concerns during the different stages of the research, they encouraged me daily.

I feel grateful towards my family, my mother, father and brother, who always believed in me and supported me to fulfil my dreams. Even though they were far away, they always felt next to me by supporting me in difficult times.
Last but not least, I would like to thank, Marios Filippoupolitis, who stood by my side throughout this PhD and inspired me with new ideas. Without him, I would not have had the courage to embark on this journey in the first place.
# GLOSSARY OF TERMS

**Latin Capitals**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>A&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Loaded cross-section of the i masonry specimen</td>
</tr>
<tr>
<td>B</td>
<td>Specimen’s width</td>
</tr>
<tr>
<td>CV</td>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>E&lt;sub&gt;0&lt;/sub&gt;</td>
<td>Initial Young’s modulus</td>
</tr>
<tr>
<td>E&lt;sub&gt;25%, 50%, 75%&lt;/sub&gt;</td>
<td>Young’s modulus at 25%, 50% and 75% of the number of loading cycles to failure</td>
</tr>
<tr>
<td>E&lt;sub&gt;95%&lt;/sub&gt;</td>
<td>Young’s modulus at 95% of the number of loading cycles to failure</td>
</tr>
<tr>
<td>F&lt;sub&gt;i,max&lt;/sub&gt;</td>
<td>Maximum load applied for the i masonry specimen</td>
</tr>
<tr>
<td>H</td>
<td>Specimen’s height</td>
</tr>
<tr>
<td>K</td>
<td>Stress intensity factor</td>
</tr>
<tr>
<td>K&lt;sub&gt;max&lt;/sub&gt;</td>
<td>Maximum stress intensity factor during a fatigue stress cycle</td>
</tr>
<tr>
<td>K&lt;sub&gt;min&lt;/sub&gt;</td>
<td>Minimum stress intensity factor during a fatigue stress cycle</td>
</tr>
<tr>
<td>L</td>
<td>Probability of survival</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformers</td>
</tr>
<tr>
<td>N</td>
<td>Number of loading cycles</td>
</tr>
<tr>
<td>N&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Number of cycles at failure</td>
</tr>
<tr>
<td>N&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Total number of loading cycles</td>
</tr>
<tr>
<td>P</td>
<td>Maximum applied load</td>
</tr>
<tr>
<td>P&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Probability of failure</td>
</tr>
<tr>
<td>R</td>
<td>Minimum to maximum stress ratio</td>
</tr>
<tr>
<td>Ra</td>
<td>Average roughness</td>
</tr>
</tbody>
</table>
Experimental and analytical investigations of brick masonry under compressive fatigue loading

\[ S \] Stress

\[ S_a \] Normalised stress amplitude ratio

\[ SD \] Standard deviation

\[ S_m \] Normalised mean stress ratio

\[ S_{\text{max}} \] Maximum applied normalised stress ratio

\[ S_{\text{min}} \] Minimum applied normalised stress ratio

\[ T \] Duration of a loading cycle

\[ T_f \] Time to failure

**Latin Lowercase**

\[ a \] Crack length

\[ e \] Eccentricity

\[ f \] Frequency

\[ f_{25\%,50\%,75\%} \] Capacity at 25%, 50% and 75% of the number of loading cycles to failure

\[ f_b \] Quasi-static compressive strength of bricks

\[ f_c \] Quasi-static compressive strength

\[ f_i \] Compressive strength for the i masonry specimen

\[ l \] Distance between centres of the outer support rollers in three point bending tests

\[ n_i \] Cumulative number of loading cycles at the i time step

\[ t \] Duration of alternating load in hours
**Greek Capitals**

- $\Delta K$: Stress intensity factor range
- $\Delta \sigma$: Stress amplitude
- $\Delta S$: Normalised stress range ratio with respect to masonry’s compressive strength

**Greek Lowercase**

- $\alpha_0$: Initial crack length
- $\alpha_{cr}$: Critical crack size
- $\varepsilon$: Strain
- $\varepsilon_0$: Initial strain
- $\varepsilon_{1-2}$: Strain at the intersection point of stage I and II
- $\varepsilon_{2-3}$: Strain at the intersection point of stage II and III
- $\varepsilon_{el}$: Elastic strain
- $\varepsilon_f$: Strain at failure
- $\varepsilon^*_{i}$: Rate of strain at the I stage of fatigue
- $\varepsilon_{\text{max}}$: Maximum strain
- $\varepsilon_{\text{min}}$: Minimum strain
- $\varepsilon_{\text{pl}}$: Plastic strain
- $\varepsilon_{\text{tot}}$: Total longitudinal strain
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f$</td>
<td>Flexural strength</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$</td>
<td>Maximum stress</td>
</tr>
<tr>
<td>$\sigma_{\text{mean}}$</td>
<td>Mean stress</td>
</tr>
<tr>
<td>$\sigma_{\text{min}}$</td>
<td>Minimum stress</td>
</tr>
</tbody>
</table>
CHAPTER ONE

1. INTRODUCTION

1.1 Background

Masonry arch bridges are an important part of the transport infrastructure in the UK and beyond (McKibbins et al., 2006; UIC code 778-3R, 2011). Their functional and economical value is undeniable, since they constitute 40% of the total European bridge stock (Page, 1994). In the UK, masonry arch bridges constitute a large percentage of the total bridges in the canal, railway and road networks (Hughes, 1996) (Figure 1-1). However, 60% of these bridges are more than a century old (Bell, 2004) and, even though they were built to meet the requirements of the era, they are now required to serve the demands of modern traffic (Figure 1-2). This means they are now exposed to increased live load and dynamic effects. Therefore, inspection, monitoring and appropriate assessment of their carrying capacity and residual life becomes of vital importance when dealing with the management of existing bridges and infrastructure (McKibbins et al., 2006; UIC code 778-3R, 2011).

![Figure 1-1 Distribution of bridge by type and ownership in the UK.](image-url)
Experimental and analytical investigations of brick masonry under compressive fatigue loading

The origins of the arch as a structural form can be traced back to antiquity (Melbourne, 2008), as excavations have revealed the remains of masonry arch structures in Mesopotamia dating back to 4000 BC (Page, 1994). The Romans realised the potential of voussoir arches to span long distances and became the first to master the construction of arch bridges to reassure efficient and permanent communications within the vast empire (Bennett, 2008). During the Medieval ages and the Renaissance, the construction techniques improved and by the middle of the 18th century the masonry arch bridge construction comes to zenith by building very flat arches supported on slender piers.

Figure 1-2 Little Stanney bridge No 141 over the Shropshire Union canal (Biggs, 2012)¹

¹ “Republished from Boatlife: continuously cruising (http://boatlife.blogspot.co.uk), Biggs, L., Copyright (2012) with permission of Biggs, J.”
There are approximately 40,000 masonry arch bridges in the UK (Tilly, 2002) and the majority was constructed between 1760 and 1900 as first the canal, then the railway, and finally the road networks were subject to rapid development and expansion (McKibbins et al., 2006). The beginning of the Canal age in the UK is considered to be 1761 with the opening of the Bridgewater canal (Hadfield, 1969; Rose & Walker, 2014). A total of 4,100 miles of canal was constructed (Figure 1-3) and the need to bridge the sundered roads emerged (McFetrich, 2010). Single-span brick or stone bridges with spans of 5-7m were initially constructed (McKibbins et al., 2006) and later aqueducts were built to carry the waterways high over rivers and roads (McFetrich, 2010). The main development of this era was the concept of mass production of arch bridges, which resulted in the standardisation of construction and materials (McKibbins et al., 2006).

Figure 1-3 Waterways of England and Wales (Canal & River Trust, 2011).²

² “Republished from https://canalrivertrust.org.uk/ with permission of the Canal and River Trust, UK.”
Expansion of the rail system in the UK took place between 1825 and 1900. During this era, many new multi-span bridges and viaducts, typically of brickwork were constructed, characterised by high-rise arches and very long spans (McKibbins et al., 2006). The heavy steam engines and longer goods trains imposed larger stresses on bridge structures and, therefore, bridges had to be stronger and more rigid in construction (Bennett, 2008). The requirements for larger spans increased with the arrival of the automobile and masonry gradually gave way to iron, steel and later to concrete.

Masonry arch bridges have been subjected to increased loading over time without regard for the structural response or deterioration due to loading, age and environmental effects (Laman et al., 2000). Bridges built prior to the late 19th century served live traffic loads consisted of no more than pedestrians, herds of animals, horses and carts, and were insignificant compared with the self-weight of the bridge (Ryall, 2008). However, live loading has changed greatly since then in terms of weights and speeds. For example, the weight of railway locomotives in UK has increased from <10 tons in 1825 to more than 120 tons in 2010 (Figure 1-4a and b) and their speed has also increased from 8.05 km/h to 300 km/h, respectively (Hayward, 2011). The advent of heavy freight wagons and high speed trains (exceeding 125 mph) now means that repetitive loading will become an important issue in the future for the design and upkeep of existing bridges (Hayward, 2011).
The oldest masonry arch bridges still in operation in the UK are the canal bridges that currently serve modern highway traffic loading. The numbers of vehicles on the roads increased, as did their speed and their weight (Ryall, 2008). The total number of licenced vehicles in the UK has increased from 143,000 in 1909 to 36,467,000 in 2015 according to the Department of Transport Statistics (Department for Transport, 2016) (Figure 1-5). In addition, the miles travelled by vehicles in total in the UK has increased from 28.9 billion miles in 1949 to 317 billion miles in 2015 (Department for Transport, 2016) (Figure 1-6). It is noteworthy that even though the increase in terms of number of vehicles and miles travelled is minor for heavy goods vehicles, the maximum permitted lorry load has increased greatly from 12 tonnes in 1904 (Ryall, 2008) to 44 tonnes in 1996 (European Communities, 1996) (Figure 1-7). These data indicate the increased dynamic loading that masonry arch bridges suffer and the need to investigate the deterioration of these structures under cyclic loading.

Figure 1-4 Changes in locomotive (a) weight and (b) axle load with time for the UK (Hayward, 2011)³

³ "Republished with permission of Taylor and Francis Group from The International Journal for the History of Engineering & Technology, 81/2, Hayward, A., Train loads on bridges 1825 to 2010, 159-191, Copyright (2011), permission conveyed through Copyright Clearance Centre Inc".
Figure 1-5 Licensed vehicles by type since 1909 in semi-logarithmic scale created from data (Department for Transport, 2016)⁴

Figure 1-6 Vehicle miles by vehicle type in Great Britain since 1949 (Department for Transport, 2016)⁵

⁴ "Republished with permission of the Department for Transport, UK."
⁵ "Republished with permission of the Department for Transport, UK."
Masonry arch bridges are assessed based on the Modified MEXE method (Department of Transport, 2001), which does not account for dynamic effects and the majority of research on masonry arch bridges, to date, has been directed towards the determination of the ultimate static loads that masonry arch bridges can carry. However, several researchers (Clark, 1994; BD 21/01, 2001; Ronca et al., 2004; Melbourne et al., 2004; Roberts et al., 2006) have recognized the need to establish serviceability limits for masonry arch bridges, in order to safeguard against progressive damage and to ensure continued safe performance. The need for establishing serviceability limits is greatly supported by recent experimental works on small-scale masonry arch models. The experimental results have shown the possibility of fatigue failure under cyclic loading at normal service level of loading, much below the ultimate load (Melbourne et al., 2004).

Figure 1-7 Variation of heavy vehicle load with time (Ryall, 2008;)

6 “Republished with permission of ICE Publishing from ICE Manual of Bridge Engineering, 75, Ryall, M.J., Loads and load distribution, 23-48, Copyright (2008), permission conveyed through Copyright Clearance Centre Inc".
Several researchers have carried out experimental tests on masonry prisms under compressive fatigue loading (Abrams et al., 1985; Clark, 1994; Ronca et al., 2004; Roberts et al., 2006; Tomor & Verstrynge, 2013) to investigate the fatigue behaviour of masonry and to establish serviceability limits for masonry arch bridges. However, doubts remain about the prediction of service load, above which accumulative damage due to cyclic loading leads to failure. Existing guidelines (Department of Transport, 2001) on the assessment of masonry arch bridges suggest loads below 50% of the ultimate failure load impose no lasting damage to the structure and the load-deflection response remains approximately linear. However, experimental studies on multi-ring masonry arch barrels under long-term cyclic loading, carried out by Melbourne et al. (2004), indicated a fatigue capacity of 37-57% of the static load carrying capacity. This study also revealed that high-cycle loading (over 10^6 cycles of loading) influences the mode of failure of a multi-ring masonry arch bridge. Hence, instead of the classical four-hinge mechanism, all multi-ring masonry arches within the test series, failed by ring separation. The formation of the hinged-mechanism is a consequence of the failure of the material under compression. On the other hand, the ring separation mechanism is linked to shear behaviour of the brick-mortar joints along the ring.

The experimental data available on the fatigue behaviour of masonry arch bridges under high-cycle loading are rather limited due to the long-term nature of testing procedures and are primarily under compression. Furthermore, minimal information is available on the fatigue behaviour of masonry under shear. Research until now has focused exclusively on the development of S-N curves and no information is available on the evolution of strain and modulus of elasticity during the fatigue life of masonry. Thus, further research on the high-cycle fatigue performance of masonry is needed to provide data for various masonry
types and masonry arch bridge failure modes. The experimental data provide information for the identification of serviceability limits and the quantification of the remaining service life of the material. Evaluation of the evolution of strain and stiffness with the loading cycles are important aspects of fatigue that need to be investigated. Such studies would allow engineers and bridge owners to identify the residual service life and safe loading limits of masonry arch bridges in the traffic network.

1.2 Research aim and objectives

The aim of this research is to identify the influence of compressive fatigue loading on the behaviour and mechanical properties of low-strength brick-masonry. This will be achieved through the following objectives:

1. Evaluate current knowledge on the fatigue deterioration of masonry
2. Generate experimental data on the response of masonry under quasi-static and fatigue compressive loading
3. Study the change in the stress-strain curves during fatigue deterioration and relate them to relevant studies for concrete
4. Propose a mathematical model for S-N-P (Stress-Number of cycles-Probability of survival) relationships of low-strength masonry during compressive fatigue loading
5. Propose a mathematical model for the evolution of strain for low-strength masonry during compressive fatigue loading
6. Propose a mathematical model for the evolution of Young’s modulus for low-strength masonry during compressive fatigue loading.

1.3 Structure of the thesis

The organisation of the thesis and the links between the different stages of research is schematically illustrated in Figure 1.8. The thesis is organised into six chapters to address all the objectives set beforehand.

<table>
<thead>
<tr>
<th>CHAPTER 1: INTRODUCTION</th>
<th>Objective 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER 2: REVIEW OF LITERATURE</td>
<td>Objectives 2 &amp; 3</td>
</tr>
<tr>
<td>CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY</td>
<td>Objectives 4, 5 &amp; 6</td>
</tr>
<tr>
<td>CHAPTER 4: EXPERIMENTAL RESULTS</td>
<td></td>
</tr>
<tr>
<td>CHAPTER 5: ANALYSIS</td>
<td></td>
</tr>
<tr>
<td>CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1.8 Outline of the thesis structure.

Chapter 1 presents the background of research and sets the aim and objectives addressed in the Thesis. In Chapter 2, a review of the existing knowledge, relevant to this research is presented. The review provides insight into the physical phenomenon of fatigue, in general, in terms of different deterioration stages and analytical procedures developed to describe
the phenomenon. Subsequently, fatigue in brittle materials is addressed and more specifically, the S-N (Stress-Number of cycles), E-N (Modulus of elasticity - Number of cycles) and ε-N (Strain-Number of cycles) relations developed for concrete under cyclic loading are reviewed. In addition, the behaviour of masonry under quasi-static and cyclic compressive loading is reviewed, together with previous experimental and analytical studies.

In Chapter 3, the methodology for the laboratory tests and analysis of the data is described. The instrumentation, monitoring techniques and data acquisition system for experimental testing, as well as the materials, type of prisms and experimental procedures are presented and justified.

Experimental data collected in the laboratory are presented in Chapter 4. More specifically, the results from quasi-static tests on clay bricks and mortar cubes under compression are described and illustrated through selected tables and figures. Additionally, the results of quasi-static and long-term fatigue tests under compression of masonry prisms are presented and discussed.

Analysis of the experimental data is presented in Chapter 5. First, the data collected during long-term fatigue tests on masonry are used to develop S-N-P (stress - number of cycles - probability) curves by adapting models used for concrete under cyclic loading and the resulting model compared with available alternative mathematical expressions. Subsequently, the ε-N (strain - number of cycles) and E-N (modulus of elasticity - number of cycles) evolution laws are described and prediction models introduced. At the end of the Chapter findings of the research are discussed and set within a wider frame.
Finally, Chapter 6 summarises the findings and highlights the conclusions from the research. The contribution of the research to knowledge is outlined, limitations are recognised and recommendations for future research provided.
CHAPTER TWO

2. REVIEW OF LITERATURE

2.1 Introduction

This chapter examines the current stage of knowledge on the effect of fatigue on the structural behaviour of masonry. An overview of fatigue is presented in Section 2.2, within which, information is provided on the definitions related to the fatigue phenomenon and the historical background. The study of material fatigue as physical phenomenon is also introduced. Subsequently, the influence of fatigue in brittle materials, focusing mainly on concrete, is reviewed. Current knowledge on the behaviour of masonry under quasi-static and long-term fatigue loading under compression is examined in Section 2.4.

2.2 Overview of fatigue

2.2.1 Definitions

Fatigue is the process of progressive localized permanent structural change occurring in a material subjected to conditions that produce fluctuating stresses and strains and that may lead to cracks and subsequent fracture after a sufficient number of loading cycles (Campbell, 2012). Repeated loading can cause microscopic physical damage that leads to the development of a crack and eventual failure, even at stresses below the ultimate strength of the material (Dowling et al., 2013). In general, lifetime is measured using the number of loading cycles to failure $N_f$. For each loading cycle, damage accumulates until
eventual failure. Evaluation of the accumulated damage allows the determination of the residual life of a structure (Bathias & Pineau, 2013).

Constant amplitude fatigue loading involves cycling between constant minimum ($\sigma_{\text{min}}$) and maximum stress levels ($\sigma_{\text{max}}$) (Dowling et al., 2013). Cyclic loading can be described by the mean ($\sigma_m$) stress and the stress amplitude ($\sigma_a$) or by the stress range ($\Delta \sigma$) and the stress ratio ($R$) (Figure 2-1). Completely reversed cycling refers to the case that $\sigma_m$ is zero and is defined by the amplitude or the maximum stress level. However, to fully define the configuration of loading the period $T$, representing the duration of a loading cycle or the frequency $f$ ($f=1/T$) presenting the number of cycles per second, is required. The relationships between the basic characteristics of cyclic loading are given in Table 2-1.

![Figure 2-1 Configuration of cyclic loading](image-url)
Table 2-1 Relationships between basic characteristics of cyclic loading

<table>
<thead>
<tr>
<th>Stress Amplitude</th>
<th>( \sigma_\alpha = \frac{\Delta \sigma}{2} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Stress</td>
<td>( \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} )</td>
</tr>
<tr>
<td>Maximum Stress</td>
<td>( \sigma_{\text{max}} = \sigma_m + \sigma_\alpha )</td>
</tr>
<tr>
<td>Minimum Stress</td>
<td>( \sigma_{\text{min}} = \sigma_m - \sigma_\alpha )</td>
</tr>
<tr>
<td>Stress Ratio</td>
<td>( R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} )</td>
</tr>
</tbody>
</table>

High-cycle fatigue involves high frequencies, low amplitudes, nominally elastic cyclic behaviour and large numbers of cycles \((N_i > 10^5)\) (Nicholas, 2006). High-cycle fatigue data are usually plotted in the form of stress \((S)\) versus the number of cycles to failure \((N_f)\) in semi-logarithmic scale (Campbell, 2012) (Figure 2-2a). The fatigue life is the number of cycles to failure at a specified stress level, while the fatigue strength is the stress at which failure does not occur at a predetermined number of cycles and the endurance limit is the stress below which failure will not occur for indefinite number of cycles (Campbell, 2012). The concept of S-N curves was developed based on the fracture mechanics domain of structural engineering (Zanuy, 2008). Another way to represent the resistance of a material against fatigue is through Goodman curves that predict the maximum sustained stress for a given number of cycles (Figure 2-2b).
2.2.2 Historical background to fatigue

Scientific studies on the fatigue of materials can be found since the 19th century, during the industrial revolution of Europe. Failure of locomotives metal components under cyclic loading triggered investigations into fatigue. The earliest article on fatigue was published in 1837, by Albert (1837), who performed the first known fatigue tests and established a correlation between the cyclic load and durability of metal. However, the first person to use the term ‘fatigue’ for metal failure due to repeated loading was Poncelet in 1839 (Bhat & Patibandla, 2011). Soon after, Rankine (1842) recognised the importance of stress concentrations in his investigation of railroad axle failures and recognized the distinctive characteristics of fatigue fractures.

Systematic fatigue testing on railway axles was undertaken by Wöhler in 1858-1870 (Wohler, 1860). Wöhler’s studies involved bending, torsion and axial loading and included fatigue tests on full-scale railway axles (Suresh, 1998). This research led to the conclusion that cyclic stress range is more important than peak stress and introduced the concept of
endurance limit, the fatigue limit representing the stress level below which the component would have infinite or very high fatigue life (Schutz, 1996). The work of Wöhler led to the development of the S-N curves (stress-number of cycles) later called Wöhler curves.

Gerber in 1874 started developing methods for fatigue design and contributed to the development of methods for the calculation of fatigue life for different stress levels (Suresh, 1998). In 1886, Bauschinger wrote the first known paper on cyclic stress-strain behaviour of materials (Bauschinger, 1886). Bauschinger, concluded that repeated stress cycles lead to change of the elastic limit, as a result of the microscopic stress distribution of the material. This phenomenon is known as the ‘Bauschinger effect’.

Ewing and Humfrey (1903) identified the stages of fatigue crack initiation and propagation in iron by the formation of slip bands using optical micrographs of cyclic damage on the specimen surface. The slip bands thicken to nucleate micro-cracks that can propagate under fatigue loading (Soboyejo, 2002). In 1910, Basquin proposed empirical laws for the S-N curves of metals and plotted Wöhler’s test data on a log-log scale (Basquin, 1910). This plot resulted in a simple linear relationship called Basquin’s law, which is still in use today.

In the following years fatigue evolved to a major field in scientific research and a wealth of scientific articles and books were published. Important was the contribution of Palmgren (1924) and Miner (1945) who developed damage accumulation models for fatigue failure. According to Palmgren (1924), applying \( n_i \) times a loading cycle with a stress amplitude \( S_{a,i} \) and a corresponding fatigue life \( N_i \) is equivalent to consuming a portion of \( n_i/N_i \) of the fatigue life. Failure occurs when 100% of the fatigue life is consumed (Equation 2-1):
Equation 2-1 is known as Miner’s rule or linear cumulative damage hypothesis. Miner (1945) studied the crack initiation life of small specimens tacitly assuming that fatigue life until failure could be considered approximately the crack initiation life. Miner (1945) also introduced the idea that fatigue damage is the consequence of work absorbed by the material, which was assumed to be proportional to the number of cycles (Schijve, 2003).

Coffin (1954) and Manson (1954) studied independently the low-cycle fatigue and concluded that macro plastic deformation occurs in every cycle and is responsible for cyclic damage (Schijve, 2003). They proposed an empirical equation, known as the Coffin-Manson equation, which relates the number of load reversals until failure with the plastic strain amplitude (Suresh, 1998).

Attempts were made using linear elastic fracture mechanics to describe the growth of fatigue cracks. Paris et al. (1961) were the first to describe the correlation between the crack growth rate, $da/dN$, and the range of the stress intensity factor, $\Delta K$ during constant amplitude cyclic loading in the form (Equation 2-2):

$$\frac{da}{dN} = C \Delta K^m$$

Equ. 2-2

Where $C$ and $m$ are experimentally obtained constants. This relationship is the most widely used expression for modelling fatigue crack growth for small plastic deformation at the crack tip. According to Paris et al. (1961), this method does not require a detailed knowledge of the mechanisms of fatigue fracture.
New electron and optical microscopy technologies allowed a more detailed study of the deformation under fatigue loading and crack initiation mechanisms. Zappfe and Worden (1951) and, later in more detail, Forsyth and Ryder (1960) studied the phenomenon of development of striations on fatigue fracture surfaces of metals. It is also worth mentioning the work of Laird (1967) and Neumann (1969), at this point, on the relationship between plastic deformation in front of a propagating crack and the occurrence of striations.

Elber (1968) observed that the tip of a growing fatigue crack in an aluminium alloy sheet specimen could be closed even at a positive stress (tensile stress) and crack opening turned out to be a non-linear function of the applied stress. Subsequently, Pearson (1975) observed from experiments on commercial alloys, that small surface cracks grow much faster than large macro cracks at nominally similar ΔK values.

Even though the studies mentioned until now mostly focus on metallic materials, significant research has also been conducted on the fatigue behaviour of ceramics, polymers and their composites (Suresh & Brockenbrough, 1990; Suresh et al., 1990; Roeben et al., 1996). Similarly, the effect of fatigue on concrete has been widely studied in terms of SN curves, deformability and damage modelling under various loading conditions (McCall, 1958; Ople & Hulsbos, 1966; Crumley & Kennedy, 1977; Holmen, 1982; Papa, 1993; Naaman & Hammoud, 1998; Alliche, 2004; Pfister et al., 2006; Singh & Kaushik, 2006; Sima et al., 2007). European Standards were also established for different materials and provide fatigue loading models (Eurocode 1, 2002) and assessment methods for steel (Eurocode 3, 2005), reinforced concrete (Eurocode 2, 2004) and aluminium (Eurocode 9, 2011) structures subjected to cyclic loading.
2.2.3 Fatigue of materials as a physical phenomenon

Fatigue is the progressive localized permanent damage occurring in a material subjected to alternating stresses or strains, which lead to generation of cracks and finally complete fracture after a sufficient number of fluctuations. The progression of fatigue damage can be divided into the following stages (Suresh, 1998):

i. Microstructural changes

ii. Nucleation of micro-cracks

iii. Crack growth and coalescence of micro-cracks to form macro-cracks

iv. Stable propagation of the dominant macro-crack

v. Final instability and fracture

Microscopic investigations have shown that nucleation of micro-cracks occurs very early during fatigue and, in some cases, almost immediately if the applied stress exceeds the fatigue limit (the cyclic stress level below which fatigue failure does not occur) (Schijve, 2001; 2003), if such a limit exists for the material under study. The crack initiation stage, during which cracks remain invisible-occupies nearly 90% of the total cycles, while once the crack becomes visible it propagates fast (only 10% of the total cycles until failure) (Bhat & Patibandla, 2011). Therefore, fatigue life consists of two major periods: the crack initiation and the crack growth period (Figure 2-3).

Figure 2-3 Different periods constituting the fatigue life of materials.
2.3 Fatigue in brittle materials

In brittle materials, crack initiation is triggered by initial faults in the bulk of the material. Especially in materials such as concrete, stone, mortar, ceramics, amongst others, there are distributed micro-cracks along grain boundaries caused due to cooling during production, which can give rise to further cracking under external loading. Finally, the surface roughness of brittle materials can cause stress concentration and lead to atomic bond rupture (Suresh, 1998).

Fatigue fracture of brittle materials under cyclic compression is a mechanical phenomenon not dependant on the environmental conditions. During cyclic compression permanent micro-crack deformation occurs at the tip of an existing crack and residual tensile stresses are generated within the micro-crack zone upon unloading. This means that the residual tensile stresses can easily exceed the tensile strength of the material and lead to the development of a mode I crack (opening mode, due to a tensile stress normal to the plane of the crack). Crack initiation and growth take place in a direction normal to the compression axis and crack length increases with an increased number of cycles. The rate of crack growth is highly influenced by the mean stress, the stress range and the stress state (Suresh, 1998).

Crack growth in brittle materials is a much more complex process compared to ductile materials. The mechanism by which the crack grows under cyclic loading is similar to that observed under monotonic loading, while the rate of crack growth is highly affected by the stress intensity factor (Ritchie, 1999). For ceramics, the crack growth rate is given by Equation 2-3.
\[
\frac{da}{dN} = B(K_{max})^g (\Delta K)^q
\]  
Eq. 2-3

Where \(\Delta K = K_{\text{max}} - K_{\text{min}}\), \(K_{\text{max}}\) and \(K_{\text{min}}\) are the maximum and minimum stress intensity factors during a fatigue stress cycle and are given by Equation 2-4 and 2-5, respectively. \(B, g\) and \(q\) are constants and satisfy Equations 2-6 and 2-7:

\[
K_{\text{max}} = \beta \sigma_{\text{max}} \sqrt{\pi \alpha}
\]  
Eq. 2-4

and

\[
K_{\text{min}} = \beta \sigma_{\text{min}} \sqrt{\pi \alpha}
\]  
Eq. 2-5

\[
B = C(1 - R)
\]  
Eq. 2-6

\[
g + q = m
\]  
Eq. 2-7

Where \(\beta\) is a dimensional factor depending on the geometry of the specimen, \(\alpha\) is the crack length and \(\sigma_{\text{max}}\) and \(\sigma_{\text{min}}\) the maximum and minimum stresses of a fatigue cycle. \(C\) is a material scaling constant and \(R\) is the stress range.

### 2.3.1 Behaviour of concrete under long-term fatigue loading

Concrete is a heterogeneous material with initial flaws (pores, air voids and shrinkage cracks) in its bulk (Lee & Barr, 2004). These initial flaws are where failure initiates from under fatigue loading (Béres, 1974). Micro-cracks progressively increase with repeated loading and leads to changes in the macroscopic behaviour of concrete (Zanuy, 2008).

Fatigue in concrete is divided into three distinct stages (Alliche, 2004):
- Stage I: extension of pre-existing cracks and the deterioration of the adhesion between pulp and aggregates (Zanuy, 2008) until a stable state is reached (10% of the fatigue life)

- Stage II: stable crack extension, nucleation of new cracks and introduction of creep effects (80% of the fatigue life)

- Stage III: unstable crack propagation until failure (10% of the fatigue life)

A number of researchers have studied the fatigue behaviour of concrete to identify the influencing parameters and develop SN curves. The maximum applied stress is considered to be the most crucial parameter and a reduction in the maximum applied stress leads to prolonged fatigue life (Breitenbucher & Ibuk, 2006). The loading frequency effect has also been widely investigated. Medeiros et al. (2015) concluded that decreased loading frequencies lead to lower sustained loading cycles.

The most widely SN model used for concrete was developed by Hsu (1981). The model provides different expressions for high-cycle (Equation 2-8) and low-cycle fatigue (Equation 2-9).

$$S_{\text{max}} = 1 - 0.0662(1 - 0.556R) \log N_f - 0.0294 \log T, \ N_f > 1000$$  \hspace{1cm} \text{Eq. 2-8}

$$S_{\text{max}} = 1.2 - 0.2R - 0.133(1 - 0.779R) \log N_f - 0.0530(1 - 0.445R) \log T, \ N_f < 1000$$  \hspace{1cm} \text{Eq. 2-9}
Where $S_{\text{max}}$ is the maximum stress level, $R$ is the ratio $S_{\text{min}}/S_{\text{max}}$, $N_f$ the number of loading cycles to failure and $T$ is the duration of one cycle. Several limitations apply, however, to the applicability of the model proposed by Hsu (1981). The strength of concrete must be below 55 MPa, $R$ must be between 0 and 1, frequency must be between 0 to 150 cycles per second and $N_f$ must be below $2 \times 10^7$.

Another SN model that has been widely used for concrete, was developed by McCall (1958). This mathematical model takes into consideration the probability of failure ($P$) (Equation 2-10).

$$L = 10^{-aR^b (\log N^c)}$$  \hspace{1cm} \text{Eq. 2-10}

Where $a$, $b$ and $c$ are experimental constants, $L = 1-P$ is the probability of survival, and $R$ is the stress ratio.

The SN curves do not provide, however, information on the deformation evolution during fatigue and the deterioration process that causes failure. The macroscopic response to progressive micro-cracking of the material is governed by growth of deformation and reduction of stiffness (Zanuy, 2008). With the application of loading cycles, the stress-strain curve is altered (Figure 2-4) significantly indicating internal material changes. The curve which is initially concave with respect to the strain axis, subsequently, becomes linear and with further loading cycles changes to convex (Holmen, 1982). Holmen (1982) concluded that the degree of convexity increases with increased loading cycles to failure.
Total strain evolution with the loading cycles exhibits an S-shape form (Figure 2-5) reflecting the internal changes of the material. This curve consists of three distinct stages, which can be described as follows:

**Stage I** (10-15% of fatigue life) rapid increase of deformation

**Stage II** stable growth of deformation (extends up to 85% of the fatigue life)

**Stage III** deformation increases quickly until failure (Medeiros et al., 2015).

The total strain at the end of the fatigue life is generally greater than in quasi-static failure and the rate of growth of deformation during the second phase is decreasing with the maximum stress. (Holmen, 1982).

---

7 "Republished from ACI Special Publication, 75, Holmen, J.O., Fatigue of concrete by constant and variable amplitude loading, 71-110, Copyright (2006), with permission from the American Concrete Institute".
The decrease of the Young’s modulus during the fatigue life is an indication of the damage occurring in the material (Alliche, 2004). The deterioration of the modulus of elasticity with the loading cycles also exhibits an S-shape form (Figure 2-6). Great decrease is observed in the first phase of fatigue, stable decrease in a lower deterioration rate during the second phase and accelerated decrease during the last cycles of loading (Breitenbucher & Ibuk, 2006). The decrease of the stiffness is larger for lower stress levels but a common limit of 60% of the initial value of the modulus of elasticity of concrete is reached at failure. (Holmen, 1982)

---

2.4 Behaviour of masonry under quasi-static and long-term fatigue compressive loading

Masonry is a non-elastic, non-homogeneous, and anisotropic material composed of two materials with very different properties: stiffer units (e.g. bricks, stones) and relatively softer mortar (e.g. lime-mortar, concrete based mortar) in a variety of material combinations (Kaushik et al., 2007). However, despite the great variability of composing materials, in all cases, the common characteristics of masonry structures are the low tensile strength and ability to sustain only compressive forces (Lourenço, 1998). A number of researchers have studied the behaviour of masonry under compressive quasi-static loading (Edgell et al., 1990; Vermeltfoort & Pluijm, 1991; Oliveira et al., 2006; Roberts et al., 2006; Mohamad et al., 2012).

The behaviour of masonry under cyclic loading is of great importance especially for the assessment of masonry arch bridges, which are subjected to dynamic loading due to traffic. However, the effect of fatigue loading on masonry has not been widely investigated. The limited available information mainly focuses on the development of SN curves for masonry under cyclic compression (Abrams et al., 1985; Clark, 1994; Ronca et al., 2004; Melbourne et al., 2004; Roberts et al., 2006; Casas, 2009; Tomor & Wang, 2010; Tomor et al., 2013; Tomor & Verstrynge, 2013).

2.4.1 Masonry under quasi-static compressive loading

Due to the inhomogeneous nature of masonry and the presence of mortar joints, which act as planes of weakness, different properties apply in different directions. In general, the compressive strength of masonry prisms is higher than the strength of mortar and lower than the strength of units (Drysdale et al., 1994).

Under uniaxial compression, the mortar is in tri-axial compression, while the unit is in compression/biaxial tension state (Lourenço, 1998) (Figure 2-7). This is attributed to the greater deformability of the mortar, which tends to expand laterally at a greater rate than the units, while being laterally confined (cohesion and friction) by the unit-mortar interface. Therefore, shear stresses at the brick-mortar interface result in an internal state of stress with triaxial compression in mortar and bilateral tension coupled with axial compression in bricks (Kaushik et al., 2007). This stress state leads eventually to the formation of vertical splitting cracks through the units and final failure of a masonry specimen (Drysdale et al., 1994).
McNary & Abrams (1985) investigated the strength and deformation of clay masonry under uniaxial concentric compressive stress. They observed that masonry prisms start spalling at 85% of the ultimate failure load. Cracks form on the sides along the headers of the specimen at about 90% of the ultimate load and subsequently propagate to cause failure due to vertical splitting.

For the analysis of a structure, the determination of the stress-strain relationship characterising the construction material is usually required. A typical stress-strain curve for clay brickwork is presented in Figure 2-8. The material behaves linearly until about one-third of the compressive strength, followed by non-linear behaviour. At about 80% of the compressive strength, vertical splitting cracks develop in the bricks and propagate (Abrams et al., 1985). Experimental tests conducted under displacement control can provide

\[\text{Figure 2-7 Triaxial state of stress at the interface of brick and mortar of masonry}^{10}\]

---

10 “Adapted from Journal of Materials in Civil Engineering, 19(9), Kaushik, H., Rai, D. & Jain, K., Stress-strain characteristics of clay brick masonry under uniaxial compression, 728-739., Copyright (2007), with permission from the ASCE”.
additional information on the post-peak softening regime of the stress-strain curve (Lourenco, 1994). During the post-peak branch the load decreases with simultaneous increase of the recorded strain (Figure 2-8).

Figure 2-8 Typical stress-strain curve for masonry in compression, 1) typical, 2) idealised diagram, 3) design diagram (Eurocode 6, 2012)

The compressive strength of masonry prisms is affected by a number of factors, for example the strength and geometry of the constitutive materials (units and mortar). Increased mortar and unit strength generally leads to increased compressive strength of the overall prism. However, requirements for better workability and deformability of the mortar suggest that higher-strength is not the most important requirement for mortars (Drysdale et al., 1994). Furthermore, compressive strength tends to decrease with increased mortar/brick thickness ratio (Hendry, 1990). Other factors that are known to affect the compressive strength of masonry are related to workmanship and environmental effects (Hendry, 1990).
2.4.2 Masonry under long-term fatigue loading

Research on the fatigue behaviour of masonry is relatively limited. The majority of the available experimental data is related to masonry under compressive cyclic loading with minimal data on shear and tensile loading. The earliest tests on brickwork prisms were carried out by Abrams et al. (1985) aiming to investigate the mechanics of masonry prisms under repeated compressive stress. Sixty masonry prisms made of the same brick type and four different mortar types were tested under various combinations of compressive cyclic stress. Sustained forces ranged from 20-70% and alternating forces from 10-40% of the ultimate static strength. Tests were terminated after only 180 loading cycles and prisms were, subsequently, loaded quasi-statically to failure. Conclusions referred that reductions of the compressive strength of masonry occur due to repeated forces. Strength reductions were attributed to inelastic straining of mortar and accumulation of lateral tractions between mortar and brick and are influenced by the mortar strength, the amplitude of the alternating stress and the number of cycles. Specifically, specimens made with stronger mortar demonstrated an increased sensitivity to repeated forces compared to specimens fabricated with low-strength mortar.

Naraine and Sinha (1989) conducted an experimental program to study the behaviour of brick masonry under cyclic compressive loading. Results showed that the envelope curve under cyclic loading coincides with the stress-strain curve under quasi-static loading. An analytical expression was proposed for the envelope curve, the stability point curve and the common point curve (Equation 2-11). The envelope curve is given by superimposition of the peak stress-strain points under cyclic loading tests on the stress-strain curve under quasi-static loading and the common points of subsequent loading and unloading stages.
give the stability point curve. To establish the common point curve, in each cycle, loading and unloading were repeated several times until the intersection point between the two procedures stabilised at a lower bound called stability point.

\[ S_{\text{max}} = \beta \frac{\varepsilon}{\alpha} e^{1-(\varepsilon/\alpha)} \]  \hspace{1cm} \text{Eq. 2-11}

Where \( S_{\text{max}} \) is the normalised stress ratio \( (\sigma_{\text{max}}/f_c) \), \( \varepsilon \) is the normalised strain ratio and \( \alpha \) and \( \beta \) are constants depending on the loading direction.

A relevant relationship (Equation 2-12) for the envelope, stability point and common point curves for masonry under biaxial compressive cyclic loading was proposed by Naraine and Sinha (1992).

\[ \sigma = (\beta f_m) \frac{\varepsilon}{(\alpha \varepsilon_m)} \exp \left( 1 - \frac{\varepsilon}{(\alpha \varepsilon_m)} \right) \]  \hspace{1cm} \text{Eq. 2-12}

where \( \sigma \) and \( \varepsilon \) are the absolute values of stress and strain respectively, \( f_m \) is the failure stress, \( \varepsilon_m \) is the strain when the peak envelope stress is reached. This expression was updated later by Alshebani and Sinha (2000) and distinct expressions were proposed for cyclic loading normal (Equation 2-13) and parallel to the bed joints (Equation 2-14), as well as, for the common point and stability curves (Equation 2-15).

\[ \sigma = \sigma_m \left( \frac{\varepsilon}{\varepsilon_m} \right)^\beta \exp \left[ \left( 1 - \frac{\varepsilon}{\alpha \varepsilon_m} \right) \frac{\varepsilon}{(\alpha + \beta) \varepsilon_m} \right] \]  \hspace{1cm} \text{Eq. 2-13}

\[ \sigma = \sigma_m \left( \frac{\varepsilon}{\varepsilon_m} \right)^\beta \exp \left[ \left( 1 - \frac{\varepsilon}{\alpha \varepsilon_m} \right) \frac{\varepsilon}{(\alpha + \beta) \varepsilon_m} \right] \]  \hspace{1cm} \text{Eq. 2-14}
Experimental and analytical investigations of brick masonry under compressive fatigue loading

\[
\sigma = \sigma_m \left( \frac{\varepsilon}{\varepsilon_m} \right)^\beta \exp \left[ \left( 1 - \frac{\varepsilon}{\alpha \varepsilon_m} \right) \frac{\varepsilon}{\varepsilon_m} \right]
\]  
Eq. 2-15

Clark (1994) tested five course brickwork prisms to study the behaviour of masonry arches under repeated loading. The prisms were centrally loaded up to 5 million loading cycles at 5Hz frequency. Samples that did not fail after 5 million cycles were considered to have run-out and subsequently were loaded to failure under quasi-static loading. The experimental tests led to the development of SN curves for both dry and wet masonry and the identification of a fatigue limit for dry brick masonry of approximately 50% of its quasi-static compressive strength. Although this value for the fatigue limit is in accordance with the existing guidelines on serviceability limits for masonry arch bridges (Department of Transport, 2001), it is based only on the maximum cyclic stress for centrally loaded tests. Thus, the effects of the induced stress range and load eccentricity (as in the case of masonry arch bridges) are not taken into account. In addition, the fatigue limit suggested by Clark (1994) refers only to samples that failed up to 5 million cycles of loading.

A series of laboratory tests were performed by Alshebani and Sinha (1999) on brickwork panels subjected to uniaxial cyclic loading in tension and compression. The authors concluded that failure in tension occurred in a brittle manner and after a limited number of loading cycles. Failure occurred by separation of the bed joints when the load was applied perpendicularly or by a zig-zag pattern when the specimen was loaded parallel to the bed joints. The failure mechanism for compressive cyclic loading was characterised by splitting in the bed joints for load application parallel to the bed joints. For load application normal to the bed joints, a combined failure of the bricks and head joints was observed. This mechanism was often accompanied by through-splitting of the midsection of the specimens. Alshebani and Sinha (1999) provided a new set of equations to describe the
envelope curves for masonry under cyclic loading normal (Equation 2-16) and parallel (Equation 2-17) to the bed joints, as well as the stability point and the common point curves (Equation 2-18). The three types of curves for brick masonry are illustrated in Figure 2-9.

\[ S = \varepsilon^\beta \exp \left( 1 - \frac{\varepsilon}{\alpha} \right) \]  
Eq. 2-16

\[ S = \varepsilon^\beta \exp \left[ (1 - \varepsilon) \frac{\varepsilon}{(\alpha + \beta)} \right] \]  
Eq. 2-17

\[ S = \varepsilon^\beta \exp \left( 1 - \frac{\varepsilon}{\alpha} \right) \varepsilon \]  
Eq. 2-18

Where \( S \) and \( \varepsilon \) are the normalised stress and strain ratios, respectively, and \( \alpha, \beta \) are constants depending on the loading direction. Alshebani and Sinha (1999) also concluded that the peak stress of the stability point curve can be considered as the maximum permissible stress level and that the plastic strain is essential on the evaluation of the permissible stress level of masonry under cyclic loading.

Figure 2-9 Envelope, stability point and the common point curves for brick masonry loaded normal and parallel to the bed joints (Alshebani, 2013)\(^{11}\).

\(^{11}\) “Reprinted from the Journal of Civil Engineering and Architecture, 7/2, Alshebani, M., Permissible stress level of brick masonry under compressive cyclic loading, 153-157, Copyright (2013), with permission from David Publishing”.  

34
Alshebani and Sinha (2001) studied the stiffness deterioration of brick masonry under cyclic compressive loading. According to the authors, stiffness and strength deteriorate with increased loading cycles and are influenced by the intensity of the cyclic loading. The stiffness ratio decreased gradually as the envelope strain increased and the deterioration exhibited a sharp decrease at lower values of the plastic strain. It was also found that the material stiffness remains stable for load ratios below 20% of the failure load and above this points stiffness commences to deteriorate. Thus, linear elastic analysis can be performed for load ratios below 20%.

Roberts et al. (2006) carried out a series of quasi-static and high cycle fatigue tests on brick masonry test specimens to investigate the serviceability limits for brick masonry arch bridges. Three types of masonry specimens were used for the quasi-static and the high-cycle fatigue tests (Figure 2-10). The specimens were fabricated using two different mortar types (IV and V as designated according to BS 5628: 1992) and tested up to seven million cycles at a loading rate of 5 Hz under different saturation degrees (dry, wet or submerged) and different load eccentricity ratios e/d (from 0 to 0.256 m).

Based on the results, Roberts et al. (2006) defined a lower-bound fatigue strength for dry, submerged and wet brick masonry as (Equation 2-19):

\[
F(S) = \frac{(\Delta \sigma \sigma_{max})^{0.5}}{f_c} = 0.7 - 0.05 \log N
\]

Eq. 2-19

Where F(S) is the function of the induced stress, \( \Delta \sigma \) is the stress range, \( \sigma_{max} \) is the maximum stress, \( f_c \) is the quasi-static compressive strength for masonry under similar loading conditions and N is the number of loading cycles.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 2-10 Three types of masonry prisms used by Roberts et al. (2006)\textsuperscript{12}.

Wang et al. (2013), after reprocessing the reported test data, suggested that the above equation may not be a true lower bound for all cases, since it is based on all the results without dividing them into subcategories according to mortar and specimen type. However, the study indicated the influence of the fatigue strength of brick masonry by the stress range, the mean or maximum induced stress and the quasi-static compressive strength.

Ronca et al. (2004) also studied the high-cycle fatigue behaviour of brick masonry under high-sustained compressive loading. The brickwork prisms were tested under concentric compressive loading at three loading frequencies (1, 5 and 10 Hz). The fatigue test data are shown in Figure 2-11, where $S_m$ is the sustained (or mean) stress applied, $S_\alpha$ the stress amplitude, and $N$ is the total number of loading cycles sustained until failure.

\textsuperscript{12}“Reprinted from Construction and Building Materials, 20/9, Roberts, T.M.; Hughes, T.G.; Dandamudi, V.R; Bell, B., Quasi static and high cycle fatigue strength of brick masonry, 603-614, Copyright (2006), with permission from Elsevier”.

36
Ronca et al. (2004) concluded that reducing the sustained load level from 80% to 65% of the average ultimate load, while maintaining the stress amplitude $S_a$ very low (5-10%), the specimens become more stable and less sensitive to fatigue loading. However, tests that did not fail during fatigue loading (termed run out) were stopped at very different maximum numbers of cycles, as shown in Figure 2-11. For example, for $S_a = 0.05$, the test was stopped after 2.5 million cycles, but for $S_a = 0.1$ the test was stopped at only 65,000 cycles. It is, therefore, difficult to predict whether the samples corresponding to $S_m = 0.65$ would not be affected by the cyclic loading and would be able to continue carrying the

---

13 “Reprinted from Structural Analysis of Historical Constructions, 2/1, Ronca, P., Franchi, A. & Crespi, P., Structural failure of historic buildings: masonry fatigue tests for an interpretation model, 273-279, Figure 17, Copyright (2004), with permission from Taylor and Francis Group”.
fatigue loading up to the similar run out limit, i.e. more than 2 million cycles (Wang et al., 2013).

Test results by Ronca et al. (2004) indicate that for brick masonry subjected to heavy sustained loading a small variation of alternating load could change the fatigue strength significantly. The results also suggest fatigue strength depends not only on the stress range/amplitude but also on the mean stress or maximum cyclic stress, as proposed by Roberts et al. (2006).

Ronca et al. (2004) observed no splitting lesions on the front side of the specimens and that vertical cracks are better visible on the shorter sides of the prism. These cracks propagate with increasing cycles. The failure mechanisms observed under fatigue loading are similar to those in Figure 2-12.

![Failure mechanisms under fatigue loading](image)

Figure 2-12 Failure mechanisms under fatigue loading (Ronca et al., 2004).<sup>14</sup>

<sup>14</sup>“Reprinted from Structural Analysis of Historical Constructions, 2/1, Ronca, P., Franchi, A. & Crespi, P., Structural failure of historic buildings: masonry fatigue tests for an interpretation model, 273-279, Figure 19, Copyright (2004), with permission from Taylor and Francis Group”.
Melbourne et al. (2004) tested a series of 3m and 5m span segmental multi-ring masonry arch barrels (Figure 2-13) under static and cyclic loading. The cyclic loading was applied at 2 Hz to represent the flow of traffic at ca. 40-50 Km/hour speed over the bridge. The tests under long-term cyclic loading indicated a fatigue capacity of 37% and 57% of the static load carrying capacity for 3 m and 5 m span arches, respectively. It was also observed that cyclic loading influences the mode of failure of the arch barrel. Instead of the classical four-hinge mechanism, all multi ring masonry arches within the test series, failed by ring separation. This effect highlights the importance of studying the shear behaviour of masonry under cyclic loading. A model for an interactive SN curve to allow assessment of residual life and fatigue performance for general arch bridge assessment was proposed.

The proposed interactive SN curve could be modified to account for rehabilitation and strengthening of the structure.

Later, Melbourne et al. (2007) proposed a new method for the assessment of masonry arch bridges based on the results from the experimental tests on masonry arch bridges. The Sustainable Masonry Arch Resistance Technique (SMART) is a holistic approach, which

---

15 “Republished with permission of ICE Publishing, from [A new masonry arch bridge Assessment strategy (SMART), Melbourne et al., 160, 2, 2007]; permission conveyed through Copyright Clearance Centre Inc.”
takes the geometry of the structure, the materials, the loading, the different modes of failure and different limit states into account. The procedure for the SMART assessment is shown in Figure 2-14. The method brings together all the existing assessment methods into a single methodology but it also considers the long-term behaviour and attempts to quantify the residual life of masonry arch bridges.

![Figure 2-14 SMART assessment procedure](image)

Bocca and Grazzini (2008) performed a series of tests to evaluate the long-term behaviour of masonry strengthening materials. Cyclic tests were performed on mixed brick-mortar specimens at 1.3 Hz frequency at 70% maximum stress level. Specimens were loaded up to 100,000 cycles and the strain-loading cycles plots revealed three distinct stages of fatigue.

---

16 “Reprinted with permission of ICE Publishing, from [A new masonry arch bridge Assessment strategy (SMART), Melbourne et al., 160, 2, 2007]; permission conveyed through Copyright Clearance Centre Inc.”
Stage I, where deformations are seen to increase rapidly (accounting for ca 10% of the service life of the test piece); stage II, where the deformations increase gradually at a virtually constant stress (10–80% of test piece life) and stage III, with a rapid increase until failure. Bocca and Grazzini (2008) calculated the variations in the deformation during stage II and through a linear regression on the logarithmic scale obtained an analytical relationship between secondary creep variations, $\frac{\partial \varepsilon}{\partial n}$ and the number of cycles ($N$) to fatigue failure (Equation 2-20). Good agreement was observed especially for the specimens that failed before 100,000 cycles.

$$N = 1839.92 \left( \frac{\partial \varepsilon_v}{\partial n} \right)^{-0.7294} \quad \text{Eq. 2-20}$$

Casas (2009) proposed a probability-based fatigue model for brick masonry in any condition (dry, wet, submerged) under compression. Casas (2009) based his analysis on the experimental data reported by Roberts et al. (2006) and developed a model that provides SN curves with different confidence levels. For example, the proposed fatigue equation for a survival probability of 95% is (Equation 2-21).

$$S_{max} = 1.106N^{-0.1034(1-R)}, \text{ for } S_{max}>0.5 \quad \text{Eq. 2-21}$$

Where $S_{max}$ is the ratio of the maximum loading stress to the quasi-static compressive strength, $N$ is the number of cycles to failure and $R$ is the ratio of the minimum stress to the maximum stress $\sigma_{min}/\sigma_{max}$. The author reports that the Weibull distribution fits the experimental data with good accuracy. However, Wang et al. (2013) suggested the model is inaccurate, as Casas (2009) was based on a few experimental results and also ignored some test data. More extensive experimental data is required in order to provide a better estimation of the parameters needed for the fatigue equation.
Tomor and Verstrynge (2013) proposed a joined fatigue-creep deterioration model. Fatigue and creep deterioration were tested within two independent laboratory test series at two universities with slightly different test setups and specimens. Tests were monitored using the acoustic emission (AE) technique. Fatigue test data were used to develop SN curves for a specific masonry type. Results indicated that a small change in stress levels leads to major changes in life expectancy. For the creep test results, the analytical relationship between stress level and time to failure proposed by Verstrynge (2010) was used (Equation 2-22).

\[
T_f = \frac{(1 - (A \cdot S + B))^{n+1}}{c(n + 1) \cdot S^n}
\]

Eq. 2-22

Where \(T_f\) is the time to failure and the parameters used were \(A = 1.9, B = -0.9, c = 8.5 \times 10^{-11}\) and \(n = 8\).

To incorporate the results in a joint model, creep loading was considered as fatigue loading with zero amplitude and static loading as fatigue failure after one cycle. Furthermore, Tomor and Verstrynge (2013) converted time to failure for creep loading into cycles to failure to allow results to be presented in one graph. For this purpose, 1 second during creep was equated to 2 cycles under fatigue loading at 2 Hz frequency. This conversion should be further examined since frequency is affecting the rate of deterioration.

Tomor and Verstrynge (2013) identified three stages of fatigue deterioration with the use of the acoustic emission technique. During the first stage (0–75% of the total number of cycles) acoustic emission was relatively low and constant. A small increase in emission was observed in the second stage (75–95% cycles), followed by rapid increase in emission and sudden failure during the third stage (95–100% cycles). In addition, a probabilistic fatigue model was proposed by adapting the model of Casas (2011) and introducing a correction.
factor C, which allows the interaction between the creep and fatigue phenomena to be taken into account and the slope of the SN curves to be adjusted (Equation 2-23).

\[
S_{\text{max}} = A \cdot N^{-B(1-C \cdot R)}
\]

Eq. 2-23

Where \( S_{\text{max}} \) is the ratio of the maximum stress to the average compressive strength (\( S_{\text{max}} = \sigma_{\text{Max}}/f_c \)), \( N \) the number of cycles, \( R \) the ratio of the minimum stress to the maximum stress (\( R = \sigma_{\text{Min}}/\sigma_{\text{Max}} \)), parameter A is set to 1, parameter B is set to 0.04 and C is the correction factor. To achieve the best correlation with the presented fatigue and creep test results, the value of 0.62 was identified for parameter C. The experimental fatigue and creep data, modified Equation 2-23 and creep model (Equation 2-22) are presented jointly in Figure 2-15.

![Figure 2-15 Joint failure model indicating stress vs. time or cycles to failure (ST or SN curves) for static, fatigue and creep loading (Tomor & Verstrynge, 2013)](image)

Figure 2-15 Joint failure model indicating stress vs. time or cycles to failure (ST or SN curves) for static, fatigue and creep loading (Tomor & Verstrynge, 2013)\(^{17}\).

\(^{17}\)“Reprinted from Construction and Building Materials, 43, Tomor, A.; Verstrynge, E., A joint fatigue–creep deterioration model for masonry with acoustic emission based damage assessment, 575-578, Copyright (2013), with permission from Elsevier”.
Tomor et al. (2013) tested small-scale brickwork specimens under quasi-static compression and fatigue loading in compression and shear. Acoustic emission monitoring was used during the tests to record the deterioration process. During the tests, the minimum stress was maintained at 10% of the strength of masonry at 2 Hz loading frequency.

Tomor et al. (2013) identified three distinct stages of fatigue based on the acoustic emission results. Stage I, during which reduction in the emission was observed, occupies the range between 0 and 32% of the total loading cycles for compression and 0-58% for shear. The emission is stabilised during the second stage (32-67% for compression, not evident in shear) and, finally, rapid increase is characterising the third stage (67-100% for compression, 58-100% shear), which leads to failure.

Carpinteri et al. (2014) performed a series of quasi-static and cyclic tests on composite masonry specimens and walls, and monitored them using the acoustic emission technique. A typical ε-N curve was obtained for masonry under fatigue (Figure 2-16), based on which three stages of fatigue were detected. Stage I during which the deformations increase rapidly for the first 10% of the fatigue life, stage II where the deformations increase at a constant rate (10-80% of the total number of loading cycles) and stage III which is characterised by rapid increase up to failure.
Carpinteri et al. (2014) proposed an equation (Equation 2-24) to relate the rate of variation of the vertical deformation during stage II, $\frac{\partial \varepsilon_v}{\partial n}$, and the number of cycles at fatigue failure $N_f$. The parameters $a$ and $b$, which are material constants, can be evaluated by applying a number of loading cycles up to the point that the deformations start growing at a constant rate. Equation 2-24 estimates the fatigue life with a good degree of accuracy.

$$N_f = a \left( \frac{\partial \varepsilon_v}{\partial n} \right)^b$$

Eq. 2-24

There is a clear need for additional test data on the fatigue behaviour of masonry under cyclic compression and especially under cyclic shear loading. Although existing guidelines

---

18 “Reprinted from Structural Control and Health Monitoring, 26/6, Carpinteri, A.; Grazzini, A.; Lacidogna, G.; Manuello, A., Durability evaluation of reinforced masonry by fatigue tests and acoustic emission technique, 950-961, Copyright (2014), with permission from John Wiley and Sons”.

45
(Department of Transport, 2001) suggest a fatigue limit for masonry of 50% of its quasi-static compressive strength, there are strong indications that it may be much lower (Melbourne et al., 2004). Thus, experimental data are needed to establish the serviceability limits for masonry arch bridges in order to safeguard the structure against progressive damage and to quantify its remaining service life by establishing reliable stress-number of cycle curves.

2.5 Chapter summary

In this Chapter, definitions on the basic aspects of fatigue loading were presented, followed by the historical background of research on this topic and the presentation of fatigue deterioration as a physical phenomenon. The behaviour of brittle materials under fatigue loading was described as an introduction for the behaviour of masonry, which is considered to be a quasi-brittle material. At the end of the chapter, research on masonry under long-term fatigue loading was presented. Through this review a wealth of knowledge on the behaviour of ductile materials in fatigue is identified. To date, a plethora of research has been performed on concrete fatigue, while very limited information is available for masonry.
CHAPTER THREE

3. RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter describes the methodology for the research on the effect of cyclic compressive loading on the properties of brick-masonry. Initially, the material selection, the specimen type and dimensions, as well as the number of tests are justified. Steps followed during the experimental studies for collecting data are discussed in detail. The instrumentation used for performing and monitoring tests, as well as, procedures used during the experimental testing of clay bricks and masonry prisms are described. Analysis of collected data in terms of SN curves, strain evolution curves and stiffness evolution curves is subsequently discussed. Development of mathematical equations to allow modelling of the behaviour of masonry under long-term fatigue loading in compression is discussed at the end of the chapter.

3.2 Outline of the research

Due to the nature of the research, a positivist approach has been adopted. Positivism requires that knowledge is derived using scientific methods, based on sensory experience gained through experiments or comparative analysis. Positivism aims at developing a unique description of a natural aspect regardless of the observer (Walliman, 2011). An experimental strategy was selected for collecting the required data, followed by a suite of
quantitative research methods to analyse the data and achieve the research objectives. Links between the different stages of the research are presented in Figure 3-1.

---

**Figure 3-1** Flow chart of research activities and links between different research methods used in this work.
3.3 Experimental Design

Before the outset of the experimental campaign that took place within this research, decisions on the selection of materials, specimen types, number of tests and procedures to be followed were made according to the needs of the research. The experimental design is presented and the choices justified.

3.3.1 Materials

The research is focusing on the long-term fatigue deterioration of masonry in connection with the longevity of masonry arch bridges. Thus, the materials selection is based on typical masonry types that comprise existing masonry arch bridges in the UK. This study is part of a wider research scheme undertaken within the University of the West of England. To represent bricks found in bridges in the traffic network, three modern solid brick types (B1 low-strength, B2 medium-strength and B3 high-strength) were considered for the test series. These brick types were selected as they are typical of masonry units found in waterways (B1), railways (B2) and modern bridges (B3). Properties of the selected bricks are listed in Table 3-1. However, current research is focusing on the behaviour of low-strength bricks (B1) and low-strength masonry representing masonry for waterways bridges. This selection was made as a worst-case scenario, since canal bridges are built with low strength masonry and are the oldest bridges in operation in the UK (McKibbins et al., 2006).

While it is not possible to source original bricks that are used in old bridges or clearly specify the basic brick properties for any bridge or bridge type due to the wide variation of
properties, the chosen new solid bricks were selected from brick suppliers based upon strength and uniformity. B1 bricks were handmade, low-strength bricks produced and supplied by Michelmersh Brick Holdings PLC. The dimensions of the bricks are 215mm x 102mm x 65mm and the gross dry density according to the material specifications is 1823 (kg/m³).

Table 3-1 Brick mechanical properties and dimensions (product specifications)

<table>
<thead>
<tr>
<th>Brick Name</th>
<th>Brick type</th>
<th>Initial rate of water absorption (kg/(m² min)) (SD, CV)</th>
<th>Compressive strength (N/mm²)</th>
<th>Surface roughness average Ra* (μm)</th>
<th>Density (kg/m³)</th>
<th>Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Michelmersh BD1.3 (handmade)</td>
<td>3.84 (0.58, 15%)</td>
<td>≥5</td>
<td>15</td>
<td>1823</td>
<td>215 x 102 x 65</td>
</tr>
<tr>
<td>B2</td>
<td>Wienerberger Warnham Terracotta Stock (moulded)</td>
<td>0.94 (0.11, 12%)</td>
<td>≥12</td>
<td>15</td>
<td>1920</td>
<td>215 x102 x 65</td>
</tr>
<tr>
<td>B3</td>
<td>Wienerberger Staffordshire Smooth Red (wirecut)</td>
<td>0.13 (0.02, 18%)</td>
<td>≥60</td>
<td>8</td>
<td>2270</td>
<td>215 x 102 x 65</td>
</tr>
</tbody>
</table>

*Ra: Roughness Average (μm)

Three types of mortar were also considered in conjunction with bricks B1, B2 and B3 to represent low-strength (M1), medium-strength (M2) and high-strength (M3) mortars for different types of masonry bridges in the UK. The selection was done in accordance with European standards (BS EN 998-2:2010) and the mortar types have been selected to be suitable for the selected brick types based on advice given by the Scottish lime centre. For all three mortar types 0.3 mm sharp washed sand was used with NHL 3.5 lime for the low-strength (M1) mortar and hydrated lime for medium-strength (M2) and high-strength (M3) mortar (Table 3-2).
Based on the selected brick and mortar types three main masonry types have been identified within the wider research series at the University of the West of England, to represent typical masonry types found in waterways, railway and recent bridges, respectively, around the UK (Tomor & Wang, 2010). These masonry types are characterized as low-strength (B1M1), medium-strength (B2M2) and high-strength (B3M3) according to their compressive strength (Table 3-3). Out of the selection, the current research is focussing on B1M1 type prisms representative of canal bridges.

Table 3-3 Different masonry types identified to represent different masonry arch bridges (Tomor & Wang, 2010)

<table>
<thead>
<tr>
<th>Masonry type</th>
<th>Notation</th>
<th>Brick strength (N/mm²)</th>
<th>Representing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-strength</td>
<td>B1M1</td>
<td>4-7</td>
<td>Waterway bridges</td>
</tr>
<tr>
<td>Medium-strength</td>
<td>B2M2</td>
<td>15-35</td>
<td>Railway bridges</td>
</tr>
<tr>
<td>High-strength</td>
<td>B3M3</td>
<td>70+</td>
<td>Recent bridges, blue bricks</td>
</tr>
</tbody>
</table>
3.3.2 Test specimens

Small-scale B1M1 masonry prisms were tested to identify the behaviour of masonry under quasi-static and cyclic compressive loading. The test specimens comprised stack-bond brick prisms built from full-size bricks and mortar joints according to the ASTM standards (2014). Masonry prisms were built out of five stacked B1 bricks and four 8 mm M1 mortar joints leading to total dimensions of 215 x 102 x 357 mm$^3$ (Figure 3-2).

![Figure 3-2 B1M1 masonry prism in accordance with ASTM standards used for the experimental tests (dimensions in mm).](image)

A different type of prism, including head joints is used by the European standards (British Standards Institution, 1999) (Figure 3-3). However, the ASTM prism was adopted for the research to be able to compare results with similar research studies on masonry (Abrams
et al., 1985; Clark, 1994; Oliveira et al., 2006; Roberts et al., 2006; Tomor & Verstrynge, 2013).

![Masonry prism diagram](image)

**Figure 3-3** Masonry prism in accordance with the European standards for testing the compressive strength of masonry.

To have systematic building quality, the same experienced master stonemason constructed all test specimens. Specimens were cured at room temperature for a minimum of five days following construction, stored outdoors for a maximum of six months in order to simulate practical curing conditions of masonry arch bridges (Oliveira, 2003) and acclimatised for a minimum of fourteen days at room temperature prior to testing (British Standards Institution, 2011).
3.3.3 Experimental tests

Experimental procedures and tests were, in all cases, in accordance with European standards. For quasi-static tests, the standards followed for bricks and masonry prisms were EN 772-1 (British Standards Institution, 2011) and EN 1052-1 (British Standards Institution, 1999) respectively. According to EN 772-1 a minimum of six tests on bricks is required for the determination of the compressive strength, while according to EN 1052-1 a minimum of three tests on masonry prisms is required.

For long-term fatigue tests on masonry prisms, testing procedures were designed according to the EN 1052-1 specifications and following procedures previously used by other researchers (Roberts et al., 2006; Tomor & Verstrynge, 2013; Tomor et al., 2013). A minimum of three specimens were tested for each experimental procedure described below. However, as it will be explained later, in several cases a much higher number of tests was performed due to the wide variability of the results to allow for a better estimation of the mean values.

3.4 Experimental Techniques

Quasi-static and long-term fatigue tests under compression were performed on bricks and brick masonry prisms. The principal objective of quasi-static tests is to characterize the mechanical behaviour of the materials under compressive loading from the undamaged state through the peak and post-peak behaviour. In addition, the fatigue tests provide further insight into the behaviour of masonry under long-term fatigue loading in terms of stress-number of cycles (S-N), strain-number of cycles (ε-N) Young’s modulus-number of cycles (E-N) and stress-strain (s-ε) curves. The collected data will be used, subsequently, to
formulate analytical expressions able to reproduce the response of the material under cyclic loading.

The data will also be elaborated to establish mathematical models for the S-N (stress-number of cycles) curves for representative masonry types to help quantify the life expectancy of a structure. More specifically, the findings will be used to develop S-N-P (stress - loading cycles - probability) expressions to identify the remaining service life (t) at any desired probability level. These S-N-P curves may later be used for the SMART method to produce information for the serviceability limits and residual service life of masonry arch bridges for specific masonry types.

3.4.1 Instrumentation

All the experimental procedures described below are standard routine tests, which were performed using the instrumentation available in the masonry arch bridge laboratory at the University of the West of England.

3.4.1.1 Fatigue testing equipment

A variety of different fatigue loading systems has been previously used to test masonry under long-term fatigue loading by various researchers. A Universal testing machine was used by Alshebani and Sinha (1999; 2001), Naraine and Sinha (1992) and Roberts et al. (2006). Verstrynge (Tomor & Verstrynge, 2013), used a hydraulic press for creep testing of masonry. A hydraulic press was also used by Ronca et al. (2004) for fatigue tests and by Cavalery et al. (2005) for quasi-static tests on masonry specimens.
The UWE laboratory is equipped with two 250 kN and two 50 kN servo-controlled hydraulic actuators (Figure 3-4), supplied by Servocon Systems Ltd, which were used for testing. The actuators can apply either quasi-static or long-term fatigue loading over extended periods of time. Servo-controlled hydraulic actuators were also used by Abrams et al. (1985) and Oliveira et al. (2006) for fatigue testing of masonry and by Page (1981) and Lourenco et al. (2004) for quasi-static tests on masonry specimens.

![Figure 3-4 250 kN servo-controlled hydraulic actuator used for quasi-static and fatigue testing of masonry.](image)

3.4.1.2 Monitoring

The longitudinal deflections, parallel to the direction of the loading, were monitored throughout the tests using Linear Variable Differential Transformers (LVDTs) with ±5 mm linear range and 0.07% accuracy (Tomor et al., 2013). Photos were also taken before the
initiation of the test, as well as after ultimate failure of the specimens to identify crack patterns and failure modes.

### 3.4.1.3 Data acquisition

The load cells, as well as the Linear Variable Differential Transducers, were connected to a digital controller provided by Servocon Systems Ltd, which, in turn, was connected to a computer. The HBM QuantumX data acquisition system (DAQ) was used to acquire different measurements simultaneously. The HBM software Catman Easy 3.1.11 was used for visualization and analysis of the experimental data. The software allows the user to record and store large data files of quasi-static and long-term fatigue loading. Catman Easy also provides the choice of real time processing or post-processing analysis of the data (HBM, 2005).

### 3.4.2 Experimental testing of bricks

Before commencing with the experimental tests on the selected masonry prisms, a series of quasi-static compressive tests were performed on B1 clay bricks to identify the material properties and uniformity of the bricks. Based on the test results, B1 bricks were selected as appropriate for masonry prisms representing waterways bridges.

#### 3.4.2.1 Preparation and test setup of quasi-static compressive tests

For the evaluation of the compressive strength and Young’s modulus of B1 bricks, a set of six bricks were tested under quasi-static compressive loading according to BS EN 772-1
(2011). Prior to the tests, bricks were conditioned in the laboratory for a minimum of fourteen days at room temperature. Subsequently, bricks were brushed to remove any superfluous materials and the upper and bottom surfaces were hand-ground to ensure that they were sufficiently flat and parallel (Oliveira, 2003; Oliveira et al., 2006). The specimens were carefully aligned with the centre of the load cell to obtain a uniform seating.

The bricks were tested under quasi-static compression, using a 250 KN capacity servo-controlled hydraulic actuator. Layers of 3mm plywood and 30mm thick steel plates were placed on both the top and bottom of the bricks to achieve even distribution of the load and to minimise local stress concentrations due to possible surface irregularities (Roberts et al., 2006). Deflections were monitored using two Linear Variable Differential Transformers (LVDTs). The LVDTs were positioned in the narrow sides of the specimen (Figure 3-5) and were attached using hot melt glue.

The loading jack was carefully lowered to touch the upper steel plate and the load was gently increased until failure occurred. The tests were conducted under displacement control at 10 μm/sec displacement rate (Tomor & Verstrynge, 2013) (Figure 3-5).
The compressive strength of the bricks was calculated according to Equation 3-1 while the Young’s modulus was calculated as the slope of the linear branch of the stress-strain curve recorded during the test.

\[ f_b = \frac{P}{A} \quad \text{Eq. 3-1} \]

Where \( P \) is the maximum applied load (N) and \( A \) is the loaded area (mm\(^2\)).

### 3.4.3 Experimental testing of mortar

Mortar cubes (M1) were tested under compressive quasi-static loading to provide information on the material properties. The research focusses on M1 type mortar used for B1M1 masonry prisms (made with B1 bricks), representative of masonry widely used for buildings and waterways bridges, built over 150 years ago.

To determine the compressive strength and the Young’s modulus, three 100 mm x 100 mm x 100 mm mortar cubes were tested in compression at 6 months age, according to BS EN 1015-11 (1999). The mortar cubes were carefully aligned with the centre of the load cell to ensure even distribution of the load and the test was performed under displacement control at a rate of 10 μm/sec (Tomor & Verstrynge, 2013). The load was applied gradually until failure to record the elastic, plastic and post-peak branches of the stress-strain curve.
3.4.4 Experimental testing of masonry prisms

Compressive quasi-static and long-term fatigue tests were performed on B1M1 masonry prisms to identify the material properties (compressive strength and Young’s modulus) and collect information on the response of masonry under fatigue loading.

3.4.4.1 Preparation and setup of quasi-static tests on masonry prisms

Before studying the high-cycle fatigue behaviour of masonry, full characterization of the behaviour of B1M1 masonry under quasi-static compression was necessary. A total of six B1M1 prisms were tested under quasi-static loading to obtain their compressive strength, Young’s modulus, maximum strain and stress-strain relationships. The material properties will serve as reference values for fatigue loading tests to evaluate the minimum and maximum stress levels.

Specimens were stored for a minimum of fourteen days at room temperature prior to testing. As with earlier tests, the upper and lower surfaces of the prisms were brushed to remove dust and impurities and ground to achieve smooth and parallel surfaces (Oliveira et al., 2006; ASTM, 2014). Prisms were placed between layers of 3 mm plywood and 30 mm of steel plates to ensure even load distribution and avoid stress concentration due to surface irregularities. Prisms were carefully aligned with the steel plates and load cell to ensure that the load was centrally applied.

Two LVDTs (Linear Variable Differential Transducers) were attached in the front and two to the back of the prism as shown in Figure 3-6. The LVDTs were positioned at 10 mm distance from the edges of the prism and set against wooden blocks. The vertical distance between the wooden blocks and the LVDTs was 81 mm and included two mortar joints (ca. 8 mm
each) and one brick (ca. 65 mm). Photographs of the prisms were also captured before the initiation of the test and after failure to provide details of the failure mechanisms and crack patterns.

Before the start of the tests, the loading jack was carefully lowered to touch the upper steel plate, avoiding sudden impact on the specimens. The load was increased using a displacement control at a constant rate of 10 μm/sec (Tomor & Verstrynge, 2013) so that failure occurred between 15 min and 30 min (British Standards Institution, 1999). Displacement control was used to allow for full recording of both the pre- and post-peak behaviour of the specimen and the respective stress-strain curves.

The compressive strength for each specimen was calculated using Equation 3-2, according to ASTM C1314-12 (ASTM, 2014). The Young’s modulus was evaluated as a secant modulus for the linear branch of the stress-strain curve. The mean value of the compressive strength and Young’s modulus was calculated as the mean of the recordings for the six prisms.

\[ f_i = \frac{F_{i,\text{max}}}{A_i} \]  

Eq. 3-2

Where \( f_i \) is the compressive strength of an individual masonry specimen, \( F_{i,\text{max}} \) (N) is the maximum load reached for the same prism and \( A_i \) (mm\(^2\)) is the loaded cross-section area.
3.4.4.2 Setup of long-term fatigue tests to failure on masonry prisms

A series of B1M1 masonry prisms were tested under long-term fatigue compressive loading until failure to identify the stress-number of cycles to failure curves (Figure 3-7) and investigate changes in the properties of masonry due to fatigue deterioration. The existence of a fatigue limit for masonry will be investigated to establish safe long-term serviceability limits for masonry. Additional information will be collected on observed crack patterns at failure.
The preparation of the prisms, and the set-up of the instrumentation for the high-cycle fatigue tests, were the same as previously described for the quasi-static loading tests (section 3.4.4.1). For the fatigue tests, the load was initially applied quasi-statically up to the mean fatigue load and, subsequently, cycled in a sinusoidal configuration (Figure 3-8) between the minimum and maximum loads (Ronca et al., 2004; Tomor & Verstrynge, 2013). The minimum ($S_{\text{min}}$) and maximum ($S_{\text{max}}$) stress levels were defined as percentages of the average compressive strength of the B1M1 specimens tested under quasi-static loading. The minimum stress level was set to 10% of the ultimate compressive strength for all fatigue tests to represent a worst-case scenario and at the same time avoid the hydraulic jack lifting up and causing sudden impact (Tomor et al., 2013). The minimum stress level was intended to represent a low level of dead load for a structure due to self-weight. The maximum stress level was set between 55% and 80% of the ultimate compressive strength for the individual specimens and was intended to represent live loading over a structure (e.g. to traffic loading over a bridge). Fatigue tests were performed under load control at 2

![Theoretical Wöhler or Stress – Number of loading cycles (SN) curve](image)

Figure 3-7 Theoretical Wöhler or Stress – Number of loading cycles (SN) curve
Hz frequency (i.e. 2 cycles per second) to represent the flow of traffic at ca. 40-50 Km/hour speed over the bridge (Melbourne et al., 2004). Longitudinal displacements were recorded at specified intervals during the tests and photographs were taken before the start of tests and after failure to record failure mechanisms and crack patterns.

![Sinusoidal configuration of load used for fatigue testing of masonry prisms.](image)

**Figure 3-8** Sinusoidal configuration of load used for fatigue testing of masonry prisms.

### 3.4.4.3 Setup of pilot long-term fatigue tests on masonry prisms for studying change in stiffness

A separate set of prisms was tested to identify changes in stiffness and stress-strain curves under high-cycle fatigue loading. While change in stiffness of brick masonry has already been studied by Alshebani and Sinha (2001) under low-cycle compressive loading, the current tests are focusing on high-cycle fatigue loading. No relevant previous test data are available.
**Pilot test design**

Instead of testing the specimens under fatigue loading to failure, during this set of tests, the specimens were loaded quasi-statically only up to the maximum cyclic stress level at specified intervals during fatigue loading. The stress-strain curves and Young’s modulus were evaluated for each quasi-static load test throughout the life of the specimen.

Specimen preparation and instrumentation were the same as described for the quasi-static tests (see section 3.4.4.1) but with different loading sequence (Figure 3-9 and Table 3-4). The loading jack was first gently lowered to touch the top of the steel plate and the load applied quasi-statically up to the mean fatigue stress level $\sigma_m$ using displacement control at a rate of 10 $\mu$m/sec (branch A of Figure 3-9). Cyclic loading was, subsequently applied between the minimum and maximum load levels for 1000 cycles under load control (Branch B of Figure 3-9) at 2 Hz frequency. The prisms were next unloaded (branch C of Figure 3-9) and loaded again quasi-statically up to the mean stress level to follow another 1000 cycles of loading. The procedure (Table 3-4) was repeated until failure occurred.

B1M1 prisms were loaded up to three different maximum stress levels (63%, 68% and 73%) of the average quasi-static compressive strength and with 10% minimum stress. For each maximum stress level a minimum of three specimens were tested.
Figure 3-9 Loading sequence of three distinct branches A) quasi-static, B) cyclic, C) unloading

Table 3-4 Description of the three loading branches for the final test design

<table>
<thead>
<tr>
<th>Branch</th>
<th>Type of loading</th>
<th>Description</th>
<th>Control type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Quasi-static</td>
<td>Loading up to the mean stress</td>
<td>Displacement control</td>
</tr>
<tr>
<td>B</td>
<td>Cyclic</td>
<td>1000 cycles of fatigue loading</td>
<td>Load control</td>
</tr>
<tr>
<td>C</td>
<td>Quasi-static</td>
<td>Unloading</td>
<td>Displacement control</td>
</tr>
</tbody>
</table>

For each masonry prism, the maximum number of loading cycles was recorded and photographs were taken after failure. The stress-strain curves were recorded for each quasi-static loading (Branch A) during the life of the specimens and a family of curves was generated for each specimen to identify changes in the strains and stiffness properties.
Changes in Young’s modulus will be used to develop a mathematical expression for the stiffness deterioration under fatigue loading.

### 3.5 Data analysis

Data from the experimental tests will be used to evaluate the material properties (stress-strain curves, compressive strength and Young’s modulus) for B1 bricks, M1 mortar and B1M1 masonry prims tested under quasi-static loading. Fatigue test data for masonry prisms will be used to define the SN curves (Stress-Number of cycles). Mathematical expressions used to describe the behaviour of concrete under fatigue loading (McCall, 1958) will be adapted to develop new S-N-P (Stress-strain-probability) relationships for masonry. The mathematical formula will be compared with existing probabilistic models proposed for masonry based on the Weibull distribution (Casas, 2011) and joint fatigue-creep model (Tomor & Verstrynge, 2013) and applied to experimental data published in the literature (Clark, 1994; Roberts et al., 2006; Tomor et al., 2013) to investigate the suitability of the model for different test data.

The mathematical expression for the S-N-P curves can in future be used to identify the condition and remaining service life for masonry structures, e.g. masonry arch bridges. More specifically, the findings can be fed into the SMART method to produce qualitative information for the serviceability limits and residual service life of masonry arch bridges for specific masonry types.

Strain-number of loading cycles (ε-N) curves will also be plotted to identify strain evolution during fatigue deterioration and a mathematical expression will be proposed based on models available for concrete under cyclic loading (Holmen, 1982; Zanuy, 2008). The elastic
and plastic components of strain will also be investigated. The Young’s modulus – number of cycles curves (E-N) will also be identified during fatigue deterioration and a mathematical model will be proposed based on the same principles as for the strain evolution model.

The proposed three different analytical models will allow the full description of the behaviour of masonry for all the stages of fatigue deterioration, i.e. crack initiation, micro-crack propagation, macro-crack propagation and final fracture.

### 3.6 Chapter summary

In this chapter, the research design and the methodology that will be followed within this work has been described. First, after presenting the links between the various stages of the research, the research design was presented in terms of the materials selected, specimen types and testing procedures. The instrumentation used to perform and monitor the various experimental tests was detailed. The experimental techniques adopted for testing bricks, mortar and masonry prisms under quasi-static and long-term fatigue loading were explained in detail in terms of specimen preparation, test setup and execution. The purpose of each test and the use of the collected data were described. Finally, the aim of the analysis of the experimentally collected data to assess the long-term fatigue life of masonry in terms of SN curves and deformability characteristics was introduced.
4. EXPERIMENTAL RESULTS

4.1 Introduction

This chapter presents the experimental data collected from the different experimental procedures introduced in Chapter 3. First, the results of compressive tests on B1 bricks and M1 mortar will be presented. Subsequently, the results of the quasi-static and long-term fatigue tests of masonry prisms under compression will be discussed. The data will be used to calculate the characteristic properties of the materials and to develop representative plots, which enable the visualisation of the behaviour of the specimens during testing. The failure modes are identified for each type of test to help explain the failure mechanisms. Finally, challenges and limitations encountered during the experimental procedures will be discussed and data will be contextualised with other experimental data.

4.2 Results of experimental tests on bricks and mortar

B1 bricks and M1 mortar cubes were tested under quasi-static compressive loading to identify the basic material properties, as described in Section 3.4.2 and 3.4.3. Test data collected were elaborated to calculate the required properties and plot the characteristic curves. In the following sections, negative strain values indicate shortening of the specimen due to compression.
4.2.1 Quasi-static compressive tests on bricks

Six B1 clay-bricks were tested under quasi-static compression, using displacement control of 10 μm/sec, to calculate the compressive strength, Young’s modulus and acquire the complete stress-strain curves that characterise the material behaviour under compression. The maximum compressive load, compressive strength and Young’s modulus for each prism are given in Table 4-1 together with their statistical parameters (Mean, Standard Deviation, Coefficients of Variation, Maximum and Minimum values). The mean compressive strength of B1 bricks is 4.86 N/mm² and the Young’s modulus is 186.92 N/mm².

Table 4-1 Results of quasi-static compression tests on B1 bricks (n=6)

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Maximum compressive load (KN)</th>
<th>Compressive Strength (N/mm²)</th>
<th>Young’s Modulus (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1-1</td>
<td>122.60</td>
<td>5.81</td>
<td>204.38</td>
</tr>
<tr>
<td>B1-2</td>
<td>92.26</td>
<td>4.39</td>
<td>57.14</td>
</tr>
<tr>
<td>B1-3</td>
<td>143.69</td>
<td>6.84</td>
<td>133.33</td>
</tr>
<tr>
<td>B1-4</td>
<td>85.36</td>
<td>4.06</td>
<td>106.66</td>
</tr>
<tr>
<td>B1-5</td>
<td>83.69</td>
<td>3.99</td>
<td>120.00</td>
</tr>
<tr>
<td>B1-6</td>
<td>88.89</td>
<td>4.05</td>
<td>500.00</td>
</tr>
<tr>
<td>Mean</td>
<td>102.75</td>
<td>4.86</td>
<td>186.92</td>
</tr>
<tr>
<td>SD</td>
<td>24.65</td>
<td>1.19</td>
<td>160.60</td>
</tr>
<tr>
<td>CV%</td>
<td>24.00</td>
<td>25.00</td>
<td>86.00</td>
</tr>
<tr>
<td>Min</td>
<td>83.69</td>
<td>3.99</td>
<td>57.14</td>
</tr>
<tr>
<td>Max</td>
<td>143.69</td>
<td>6.84</td>
<td>500.00</td>
</tr>
</tbody>
</table>

Recordings of the LVDTs during quasi-static compression tests were used to plot the stress-strain curves for B1 bricks (Figure 4-1). However, due to extended cracking, the LVDTs were detached from the bricks before the ultimate failure load was reached for specimens B1-1, B1-2 and B1-6 and the curves could not be fully recorded. The stress-strain curve generally
remains linear (elastic behaviour) to about one third of the ultimate strength of the brick (Figure 4-1). The behaviour, subsequently, becomes non-linear until the maximum stress. The post-peak behaviour is unstable, collapse occurs suddenly and it was not possible to be recorded for all specimens. The maximum recorded strain ranged from -0.05 to -0.08 mm/mm (only fully recorded curves are considered).

![Figure 4-1 Typical stress–strain curve for B1 bricks under quasi-static compression](image)

4.2.2 Quasi-static compressive tests on mortar

Three M1 mortar cubes were tested under quasi-static compression under displacement control of 10 μm/sec to evaluate the compressive strength, Young’s modulus and stress-strain curves. The maximum compressive load, compressive strength and Young’s modulus for each mortar cube are given in Table 4-2 together with their statistical parameters (Mean, Standard Deviation, Coefficients of Variation, Maximum and Minimum values). The
mean compressive strength of M1 type mortar is 1.09 N/mm² and the Young’s modulus is 873.33 N/mm².

Table 4-2 Results of quasi-static compression tests on hardened M1 mortar cubes

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Compressive Strength (N/mm²)</th>
<th>Young’s modulus (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1-1</td>
<td>1.07</td>
<td>1170.00</td>
</tr>
<tr>
<td>M1-2</td>
<td>1.08</td>
<td>700.00</td>
</tr>
<tr>
<td>M1-2</td>
<td>1.13</td>
<td>750.00</td>
</tr>
<tr>
<td>Mean</td>
<td>1.09</td>
<td>873.33</td>
</tr>
<tr>
<td>SD</td>
<td>0.03</td>
<td>258.13</td>
</tr>
<tr>
<td>CV%</td>
<td>3.20</td>
<td>29.55</td>
</tr>
</tbody>
</table>

A typical stress-strain curve for mortar under quasi-static compression is presented in Figure 4-2. The sharp linear branch increases to about 90% of the compressive strength, followed by an extended softening branch. The maximum recorded strain ranges from -0.025 to -0.045 mm/mm.

Figure 4-2 Typical stress-strain curve for M1 hardened mortar under quasi-static compression
4.3 Results of experimental tests on masonry prisms

B1M1 masonry prisms were tested under compressive quasi-static and long-term fatigue loading, as described in Section 3.4.4, to identify the basic properties of the material and to evaluate the effect of cyclic loading on the behaviour of masonry. Test data that were collected during the experimental testing were elaborated to calculate the required properties and address the research objectives.

4.3.1 Quasi-static compressive tests on masonry prisms

Six B1M1 masonry prisms were tested under compressive quasi-static loading as discussed in Section 3.4.4.1. The compressive strength and Young’s modulus were calculated for each prism according to (BS EN 1052-1:1999) and shown in Table 4-3. The mean compressive strength is 2.94 N/mm² and the Young’s Modulus is 447.87 N/mm². The large coefficient of variation for the Young’s Modulus is due to the graphic method used for the calculation and depends on the interpretation of the initial tangent of the curves. A similar coefficient of variation for the Young’s modulus of masonry with NHL3.5 lime mortar was observed by Costigan et al. (2015).

Figure 4-3 illustrates a typical stress-strain curve for a B1M1 masonry prism tested under compressive quasi-static loading in the direction normal to the bed joints. The curve exhibits three distinct branches. The initial short branch corresponds to the period needed for adjustment between the specimen and the loading system. Next, a linear branch extends up to approximately 0.33 of the ultimate load (f_u) after which cracks start to develop and introduce non-linearity. At about 0.75 f_u, vertical splitting cracks start developing in the bricks and propagate until the maximum stress is reached. After the
maximum load is reached, the load starts to decrease rapidly and leads to sudden collapse. The post-peak behaviour was generally very short and only a small portion of it could be recorded. The maximum recorder strain varied from -0.012 to -0.025 mm/mm.

Table 4-3 Results of quasi-static compression tests on B1M1 prisms

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Failure load (KN)</th>
<th>Compressive strength (N/mm²)</th>
<th>Young’s Modulus (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-1</td>
<td>64.30</td>
<td>3.06</td>
<td>561.09</td>
</tr>
<tr>
<td>B1M1-2</td>
<td>60.22</td>
<td>2.87</td>
<td>382.30</td>
</tr>
<tr>
<td>B1M1-3</td>
<td>61.16</td>
<td>2.91</td>
<td>640.63</td>
</tr>
<tr>
<td>B1M1-4</td>
<td>63.87</td>
<td>3.04</td>
<td>303.57</td>
</tr>
<tr>
<td>B1M1-5</td>
<td>61.03</td>
<td>2.91</td>
<td>380.97</td>
</tr>
<tr>
<td>B1M1-6</td>
<td>59.34</td>
<td>2.83</td>
<td>418.66</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><strong>61.65</strong></td>
<td><strong>2.94</strong></td>
<td><strong>447.87</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td><strong>2.00</strong></td>
<td><strong>0.10</strong></td>
<td><strong>126.83</strong></td>
</tr>
<tr>
<td><strong>CV%</strong></td>
<td><strong>3.00</strong></td>
<td><strong>3.00</strong></td>
<td><strong>28.00</strong></td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td><strong>59.34</strong></td>
<td><strong>2.83</strong></td>
<td><strong>303.57</strong></td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td><strong>64.30</strong></td>
<td><strong>3.06</strong></td>
<td><strong>640.63</strong></td>
</tr>
</tbody>
</table>

Figure 4-3 Typical stress-strain curve for B1M1 prisms under quasi-static compression
Vertical cracks first appeared in the units around the centre of the specimens, followed by further vertical cracking at the narrow sides, leading to failure. In Figure 4-4 the almost vertical cracks and swelling of the masonry prism just before failure are shown.

![Figure 4-4 Crack pattern and spalling of a masonry prism at ultimate failure under quasi-static compression](image)

**Figure 4-4 Crack pattern and spalling of a masonry prism at ultimate failure under quasi-static compression**

[a] front view and [b] narrow side of the prism.

Comparison of the stress-strain curves for bricks, mortar cubes and masonry prisms, indicated that the compressive strength for masonry lies between the compressive strength of brick and mortar as masonry is stronger than mortar and weaker than brick (Drysdale et al., 1994). However, the stiffness of B1M1 masonry is greater than the stiffness of B1 bricks and lower than the stiffness of M1 mortar. The mechanical characteristics of masonry do not always lie between the mechanical characteristics of mortar and bricks,
especially when the strength and stiffness of bricks and mortar are comparable (Kaushik et al., 2007). Additionally, the maximum recorded strain for masonry is remarkably lower than strain recorded for mortar cubes and bricks.

![Typical stress-strain curves for B1 brick, M1 mortar and B1M1 masonry prism](image)

**Figure 4-5** Typical stress-strain curves for B1 brick, M1 mortar and B1M1 masonry prism

### 4.3.2 Long-term compressive fatigue tests on masonry prisms

A series of B1M1 masonry prisms were tested under long-term fatigue compressive loading. The maximum stress level for the various specimens was 55%, 60%, 68% or 80% of the mean compressive strength of masonry $f_c$ that was calculated in section 4.3.1. A minimum of three prisms tested at each maximum stress level. The minimum stress level was kept at 10% of $f_c$ and loading frequency at 2 Hz for all the fatigue tests.

The mean number of cycles to failure and standard deviation are listed in Table 4-4 to Table 4-7 for different stress levels. Although prism B1M1-45 did not fail under long-term fatigue loading and the test was terminated after 10,225,480 cycles (approximately 2 months) without any signs of deterioration, it is also included in Table 4-4.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Table 4-4 Long-term fatigue data for 55% of the maximum compressive stress

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Max Load (kN)</th>
<th>Max Stress $\sigma_{\text{max}}$ (N/mm²)</th>
<th>Max Stress (%)</th>
<th>Min Load (kN)</th>
<th>Min Stress $\sigma_{\text{min}}$ (N/mm²)</th>
<th>Min Stress (%)</th>
<th>$R = \sigma_{\text{min}} / \sigma_{\text{max}}$</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-34</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>56562</td>
</tr>
<tr>
<td>B1M1-40</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>412,774</td>
</tr>
<tr>
<td>B1M1-41</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>1,088,560</td>
</tr>
<tr>
<td>B1M1-43</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>2,200</td>
</tr>
<tr>
<td>B1M1-44</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>4,864</td>
</tr>
<tr>
<td>B1M1-45</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>10,225,676*</td>
</tr>
<tr>
<td>B1M1-46</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>1,724,587</td>
</tr>
<tr>
<td>B1M1-47</td>
<td>34.00</td>
<td>1.62</td>
<td>55.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.18</td>
<td>1,672,237</td>
</tr>
</tbody>
</table>

*No failure

Table 4-5 Long-term fatigue data for 60% of the maximum compressive stress

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Max Load (kN)</th>
<th>Max Stress $\sigma_{\text{max}}$ (N/mm²)</th>
<th>Max Stress (%)</th>
<th>Min Load (kN)</th>
<th>Min Stress $\sigma_{\text{min}}$ (N/mm²)</th>
<th>Min Stress (%)</th>
<th>$R = \sigma_{\text{min}} / \sigma_{\text{max}}$</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-26</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>25,342</td>
</tr>
<tr>
<td>B1M1-28</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>2,646,302</td>
</tr>
<tr>
<td>B1M1-29</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>122,762</td>
</tr>
<tr>
<td>B1M1-30</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>1,268,627</td>
</tr>
<tr>
<td>B1M1-31</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>3,528,118</td>
</tr>
<tr>
<td>B1M1-32</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>986,325</td>
</tr>
<tr>
<td>B1M1-33</td>
<td>37.00</td>
<td>1.76</td>
<td>60.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.16</td>
<td>796,744</td>
</tr>
</tbody>
</table>

 Mean 1,339,174
 CV% 97.00

Table 4-6 Long-term fatigue data for 68% of the maximum compressive stress

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Max Load (kN)</th>
<th>Max Stress $\sigma_{\text{max}}$ (N/mm²)</th>
<th>Max Stress (%)</th>
<th>Min Load (kN)</th>
<th>Min Stress $\sigma_{\text{min}}$ (N/mm²)</th>
<th>Min Stress (%)</th>
<th>$R = \sigma_{\text{min}} / \sigma_{\text{max}}$</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-19</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>1,800</td>
</tr>
<tr>
<td>B1M1-20</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>3,600</td>
</tr>
<tr>
<td>B1M1-21</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>13,000</td>
</tr>
<tr>
<td>B1M1-22</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>17,350</td>
</tr>
<tr>
<td>B1M1-23</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>18,651</td>
</tr>
<tr>
<td>B1M1-24</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>18,276</td>
</tr>
<tr>
<td>B1M1-25</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>3,000</td>
</tr>
<tr>
<td>B1M1-26</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>6,737</td>
</tr>
<tr>
<td>B1M1-27</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>134</td>
</tr>
<tr>
<td>B1M1-28</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>3,541</td>
</tr>
<tr>
<td>B1M1-29</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>5,994</td>
</tr>
<tr>
<td>B1M1-30</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>212</td>
</tr>
<tr>
<td>B1M1-31</td>
<td>42.00</td>
<td>2.00</td>
<td>68.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.15</td>
<td>1,100</td>
</tr>
</tbody>
</table>

 Mean 7,184
 CV% 98.53
Table 4-7 Long-term fatigue data for 80% of the maximum compressive stress

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Max Load (kN)</th>
<th>Max Stress $\sigma_{\text{max}}$ (N/mm$^2$)</th>
<th>Max Stress (%)</th>
<th>Min Load (kN)</th>
<th>Min Stress $\sigma_{\text{min}}$ (N/mm$^2$)</th>
<th>Min Stress (%)</th>
<th>$R (\sigma_{\text{min}}/\sigma_{\text{max}})$</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-18</td>
<td>49.00</td>
<td>2.33</td>
<td>80.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.12</td>
<td>2,566</td>
</tr>
<tr>
<td>B1M1-48</td>
<td>49.00</td>
<td>2.33</td>
<td>80.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.12</td>
<td>14,073</td>
</tr>
<tr>
<td>B1M1-49</td>
<td>49.00</td>
<td>2.33</td>
<td>80.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.12</td>
<td>2,832</td>
</tr>
<tr>
<td>B1M1-50</td>
<td>49.00</td>
<td>2.33</td>
<td>80.00</td>
<td>6.00</td>
<td>0.29</td>
<td>10.00</td>
<td>0.12</td>
<td>456</td>
</tr>
</tbody>
</table>

Mean 4,981.75

CV% 123.52

The fatigue data exhibit large scatter as indicated by the large Coefficient of Variation values. The phenomenon of scatter for fatigue test data under the same loading conditions is well known and attributed to differences in the microstructure for different specimens (Xuesong, 2014). Potential sources of scatter could be the specimen production and surface quality, accuracy of testing equipment, laboratory environment and skill of laboratory technicians (Schijve, 2001). Scatter is generally larger for low stress amplitudes (Schijve, 2001). This trend is not apparent for the test data presented in the above tables as the scatter for 80% maximum stress is large. This, however, is due to the small number of tests performed at this stress level.

The fatigue data listed in Table 4-4 to Table 4-7 is shown in the form of a Stress-Number of cycles graph (maximum stress level against number of cycles at failure) in Figure 4-6. The number of cycles (N) are shown on a log scale and static test results are indicated as failure at 1 cycle. The results exhibit a notable scatter but indicate increased number of loading cycles for lower stress levels.

An endurance limit exists for steel but not for concrete. Based on the test data, an endurance limit can not be identified for masonry and if there is such a limit it is likely to be below 55% stress level. However, testing masonry prisms at stress levels below 55% at 2 Hz loading frequency would require months of lab work.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-6 Experimental data of long-term fatigue tests under compression for B1M1 type masonry (n=38)

The evolution of the maximum and minimum recorded total longitudinal strain with the loading cycles ($\varepsilon$-$N$) was also plotted (Figure 4-7 to Figure 4-10) for each maximum stress level (55\%, 60\%, 68\% and 80\% of $f_c$). The $\varepsilon$-$N$ curve resembles an S-shape with three distinct stages:

- **Stage I**: high rate of strain during the first ca. 10\% of the fatigue life due to initiation of micro-cracks.

- **Stage II**: slow and constant increase in strain between ca. 10\% and 90\% of the fatigue life due to micro-crack development.

- **Stage III**: rapid increase in strain from ca. 90\% of the fatigue life due to coalition of micro-cracks into macro-cracks, leading to ultimate failure.
Figure 4-7 Total longitudinal strain variation with the cycle ratio for 55% maximum stress level (a) maximum total strain, (b) minimum total strain
Figure 4-8 Total longitudinal strain variation with the cycle ratio for 60% maximum stress level (a) maximum total strain, (b) minimum total strain
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-9 Total longitudinal strain variation with the cycle ratio for 68% maximum stress level (a) maximum total strain, (b) minimum total strain
Figure 4-10 Total longitudinal strain variation with the cycle ratio for 80% maximum stress level (a) maximum total strain, (b) minimum total strain
Three stages of fatigue deterioration have been identified in the past for masonry (Carpinteri et al., 2014) and for concrete (Holmen, 1982; Kim & Kim, 1996; Breitenbuecher & Ibuk, 2006; Zanuy et al., 2011). According to Carpinteri et al. (2014) the duration of stage II approximates the fatigue life with good accuracy. The rate of strain development during stage II is plotted against the maximum stress level in Figure 4-11. Prism B1M1 that did not fail and prism B1M1-53 that failed too early are ignored in this plot. The rate of strain development indicates that the higher the maximum stress, the steeper the second stage of the curve appears to be. However, the scatter of data is large especially for 55% maximum stress.

The rate of strain development during stage II is compared against the loading cycles to failure in Figure 4-12 for 55%, 60%, 68% and 80% maximum stress levels. The strain development rate decreases for increased loading cycles to failure. The decrease is,
however, larger for loading cycles below 10,000. For higher loading cycles the strain development rate seems to stabilise between 0.002 and 0.003 mm/mm/cycle.

![Figure 4-12 Strain rate of stage II with loading cycles to failure for 55%, 60%, 68% and 80% maximum stress (n=30)](image)

The duration of the three stages was calculated for each prism as presented in Table 4-8. For prisms B1M1-50 and B1M1-56 no data were recorded for the early and late stages of fatigue life, due to early failure of the specimens and, therefore, the durations of the different stages could not be calculated.

The mean duration of Stage I of fatigue is 9.46% of the total loading cycles and varies between 6% and 14%. The mean value for the end of stage II is 86.14% of the total loading cycles and varies between 76% and 94%. There is no clear indication that the total loading
cycles or the maximum stress level influences the duration of each of the three stages of fatigue.

Table 4-8 Duration of different stages of fatigue for each prism

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Max Stress (%)</th>
<th>Number of Cycles</th>
<th>End of stage I (% Nf)</th>
<th>End of stage II (% Nf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-18</td>
<td>80</td>
<td>2566</td>
<td>12</td>
<td>90</td>
</tr>
<tr>
<td>B1M1-48</td>
<td>80</td>
<td>14073</td>
<td>8</td>
<td>76</td>
</tr>
<tr>
<td>B1M1-49</td>
<td>80</td>
<td>2832</td>
<td>10</td>
<td>78</td>
</tr>
<tr>
<td>B1M1-50</td>
<td>80</td>
<td>45</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B1M1-19</td>
<td>68</td>
<td>1800</td>
<td>10</td>
<td>84</td>
</tr>
<tr>
<td>B1M1-20</td>
<td>68</td>
<td>3600</td>
<td>14</td>
<td>90</td>
</tr>
<tr>
<td>B1M1-21</td>
<td>68</td>
<td>13000</td>
<td>12</td>
<td>86</td>
</tr>
<tr>
<td>B1M1-22</td>
<td>68</td>
<td>17350</td>
<td>8</td>
<td>86</td>
</tr>
<tr>
<td>B1M1-23</td>
<td>68</td>
<td>18651</td>
<td>8</td>
<td>88</td>
</tr>
<tr>
<td>B1M1-24</td>
<td>68</td>
<td>18276</td>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>B1M1-35</td>
<td>68</td>
<td>3000</td>
<td>13</td>
<td>87</td>
</tr>
<tr>
<td>B1M1-36</td>
<td>68</td>
<td>6737</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>B1M1-54</td>
<td>68</td>
<td>3541</td>
<td>10</td>
<td>82</td>
</tr>
<tr>
<td>B1M1-55</td>
<td>68</td>
<td>5994</td>
<td>14</td>
<td>78</td>
</tr>
<tr>
<td>B1M1-56</td>
<td>68</td>
<td>212</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B1M1-57</td>
<td>68</td>
<td>1100</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>B1M1-26</td>
<td>60</td>
<td>25342</td>
<td>8</td>
<td>84</td>
</tr>
<tr>
<td>B1M1-28</td>
<td>60</td>
<td>2646302</td>
<td>8</td>
<td>88</td>
</tr>
<tr>
<td>B1M1-29</td>
<td>60</td>
<td>122762</td>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>B1M1-30</td>
<td>60</td>
<td>1268627</td>
<td>8</td>
<td>94</td>
</tr>
<tr>
<td>B1M1-31</td>
<td>60</td>
<td>3528118</td>
<td>8</td>
<td>90</td>
</tr>
<tr>
<td>B1M1-32</td>
<td>60</td>
<td>986325</td>
<td>10</td>
<td>86</td>
</tr>
<tr>
<td>B1M1-33</td>
<td>60</td>
<td>796744</td>
<td>6</td>
<td>76</td>
</tr>
<tr>
<td>B1M1-34</td>
<td>55</td>
<td>56562</td>
<td>6</td>
<td>86</td>
</tr>
<tr>
<td>B1M1-40</td>
<td>55</td>
<td>412774</td>
<td>8</td>
<td>92</td>
</tr>
<tr>
<td>B1M1-41</td>
<td>55</td>
<td>1088560</td>
<td>8</td>
<td>88</td>
</tr>
<tr>
<td>B1M1-43</td>
<td>55</td>
<td>2200</td>
<td>14</td>
<td>82</td>
</tr>
<tr>
<td>B1M1-44</td>
<td>55</td>
<td>4864</td>
<td>10</td>
<td>88</td>
</tr>
<tr>
<td>B1M1-46</td>
<td>55</td>
<td>1724587</td>
<td>6</td>
<td>94</td>
</tr>
<tr>
<td>B1M1-47</td>
<td>55</td>
<td>1672237</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

The maximum, minimum and the calculated mean strain at the end of stage I, II and III are depicted in Figures 4-13, 4-14 and 4-15 respectively. The values of the strain at the
intersection points between adjacent stages will be used later to generate a model to predict strain evolution during fatigue life.

Figure 4-13 Minimum, maximum and mean recorded strains at the end of stage I for different maximum stress levels (n=30)

Figure 4-14 Minimum, maximum and mean recorded strains at the end of stage II for different maximum stress levels (n=30)
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-15 Minimum, maximum and mean recorded strains at the end of stage III for different maximum stress levels (n=30)

The relationship between the normalised total maximum longitudinal strain ratio (i.e. the strain recorded after a specific number of loading cycles over the initially recorded strain at the beginning of the test) and the total loading cycles sustained until failure is presented in Figure 4-16, 4-17 and 4-18 at the end of stage I, II and III, respectively. Strain increases up to 5.25 times the strain recorded after the first cycle \(\varepsilon_0\). The total maximum strain appears to get larger with the loading cycles at all stages of fatigue, that is likely to be caused by the increased effects of creep induced with higher numbers of fatigue cycles. At lower strain rates the total test time is extended and creep damage is accumulated during the relatively longer time spent near the peak stress of each cycle (Esztergar, 1972).
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-16 Evolution of strain with the number of cycles to failure at the end of stage I (n=30)

Figure 4-17 Evolution of strain with the number of cycles to failure at the end of stage II (n=30)
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Although during fatigue loading specimens failed at significantly lower stress levels than their ultimate quasi-static strength, failure patterns were similar to those under quasi-static loading. For all specimens failure occurred by the development of vertical cracks through the bricks and mortar joints, causing splitting of the prisms (Figure 4-19). Major cracks developed along the narrow sides and swelling of the specimens was observed.
Figure 4-19 Representative failure modes of masonry prisms under fatigue compression loading at 55%, 60%, 68% and 80% maximum stress level.
4.3.3 Pilot long-term fatigue tests on masonry prisms for studying change in stiffness

B1M1 brick masonry prisms were tested under maximum stress levels 73%, 68% and 63% and minimum stress level 10% of the compressive strength. After every 1000 cycles quasi-static loading was applied up to the maximum applied fatigue stress level to identify the stress-strain relationship during fatigue deterioration process (as described in Section 3.4.4.3).

For the maximum stress level of 73% all the prisms, however, failed very early (before 1000 cycles) and only the initial stress strain curve was recorded (Table 4-9). Therefore, no useful test data on the change of stiffness with the loading cycles were obtained at this stress level. The data can, however, be considered for the development of a model for the SN curves.

Table 4-9 Fatigue test results for 73% maximum stress level.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Min stress N/mm²</th>
<th>Min stress %</th>
<th>Max stress N/mm²</th>
<th>Max stress %</th>
<th>Number of cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-66</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>253</td>
</tr>
<tr>
<td>B1M1-67</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>200</td>
</tr>
<tr>
<td>B1M1-68</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>413</td>
</tr>
<tr>
<td>B1M1-69</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>53</td>
</tr>
<tr>
<td>B1M1-70</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>55</td>
</tr>
<tr>
<td>B1M1-76</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>7</td>
</tr>
<tr>
<td>B1M1-77</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>104</td>
</tr>
<tr>
<td>B1M1-78</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>240</td>
</tr>
<tr>
<td>B1M1-85</td>
<td>0.29</td>
<td>10.00</td>
<td>2.14</td>
<td>73.00</td>
<td>93</td>
</tr>
</tbody>
</table>

| Mean          | 157.56           |
| CV%           | 80.00            |
For 63% and 68% maximum stress levels, the numbers of cycles to failure are presented in Table 4-10 and Table 4-11, respectively. Prism B1M1-65 did not fail after 275,000 cycles (running time approximately 3 weeks) and the test had to be terminated due to time constraints. For each prism, the stress-strain curve was plotted after every 1000 loading cycles (except for prisms B1M1-83 and B1M1-88 where signs of early cracking was observed and stress-strain curves were plotted after every 500 cycles). Figure 4-20 depicts the SN relationship for the test data presented in section 4.3.2 enriched with the test data for 63%, 68% and 73% maximum stress levels.

Table 4-10 Fatigue test results for 63% maximum stress level.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Min stress N/mm²</th>
<th>Min stress %</th>
<th>Max stress N/mm²</th>
<th>Max stress %</th>
<th>Number of cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-71</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>718</td>
</tr>
<tr>
<td>B1M1-72</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>11,038</td>
</tr>
<tr>
<td>B1M1-73</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>269</td>
</tr>
<tr>
<td>B1M1-74</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>2,515</td>
</tr>
<tr>
<td>B1M1-75</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>1,104</td>
</tr>
<tr>
<td>B1M1-79</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>266</td>
</tr>
<tr>
<td>B1M1-80</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>19,203</td>
</tr>
<tr>
<td>B1M1-81</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>54</td>
</tr>
<tr>
<td>B1M1-82</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>34,728</td>
</tr>
<tr>
<td>B1M1-83</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>3,355</td>
</tr>
<tr>
<td>B1M1-84</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>256</td>
</tr>
<tr>
<td>B1M1-86</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>59,921</td>
</tr>
<tr>
<td>B1M1-87</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>543</td>
</tr>
<tr>
<td>B1M1-88</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>4,809</td>
</tr>
<tr>
<td>B1M1-89</td>
<td>0.29</td>
<td>10.00</td>
<td>1.86</td>
<td>63.00</td>
<td>881</td>
</tr>
</tbody>
</table>

| Mean | 9,310.67 |
| CV%  | 182.23   |
Table 4-11 Fatigue test results for 68% maximum stress level.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Min stress N/mm²</th>
<th>Min stress %</th>
<th>Max stress N/mm²</th>
<th>Max stress %</th>
<th>Number of cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1M1-58</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>31,000</td>
</tr>
<tr>
<td>B1M1-59</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>69,537</td>
</tr>
<tr>
<td>B1M1-60</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>34</td>
</tr>
<tr>
<td>B1M1-61</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>71,342</td>
</tr>
<tr>
<td>B1M1-62</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>11,754</td>
</tr>
<tr>
<td>B1M1-63</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>37,938</td>
</tr>
<tr>
<td>B1M1-64</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>33,752</td>
</tr>
<tr>
<td>B1M1-65</td>
<td>0.29</td>
<td>10.00</td>
<td>2.00</td>
<td>68.00</td>
<td>275,000*</td>
</tr>
</tbody>
</table>

*No failure

Mean 63,169.63

CV% 125.76

Figure 4-20 Experimental data of long-term fatigue tests under compression for B1M1 type masonry enriched with data for 63%, 68% and 73% stress levels (n=70)
Within the test data presented in Table 4-9 to Table 4-11 there are some data that could be considered anomalous. At 73% maximum stress prism B1M1 failed after only 7 cycles, at 63% maximum stress prism B1M1-81 failed at 54 cycles and at 68% maximum stress level prism B1M1-60 failed after 34 loading cycles. No problems associated with the laboratory equipment or the test setup were recorded during testing those prisms. The Coefficient of Variability calculated for the above groups of test data are comparable to the values obtained in Section 4.3.3. In Figure 4-21 the Coefficient of Variation for the fatigue test data is plotted against the maximum stress level. The Coefficient of Variation is generally increasing for lower stress levels. The Coefficient of Variation at 80% maximum stress level can be considered anomalous. This anomaly could be due to the small number of tests performed at this stress level (n=4).

![Figure 4-21 Coefficient of Variation of fatigue test data with the maximum stress level.](image)

The stress-strain data recorded at different stages of the fatigue life as well as the total maximum and minimum longitudinal strain against the number of cycles for each specimen
are shown in Figure 4-22 to Figure 4-33 (a and b) for 68% and 63% maximum stress levels. Changes in the stress-strain curves with the loading cycles indicate that residual strain increases with increased number of loading cycles (stress-strain curves shifts towards the left in the plots). The residual strain is larger at the start of the test, it becomes smaller and more constant during the majority of the loading cycles and increases rapidly just before failure. The shape of the stress-strain curve is initially straight or, in some cases, slightly concave towards the strain axis. With increased loading cycles the shape of the curve changes to convex with respect to the strain axis and becomes increasingly curved.
Figure 4-22 [a] Stress-strain curve and [b] total longitudinal strain variation for B1M1-58 at 68% maximum stress with number of cycles.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-23 Stress-strain curve and [b] total longitudinal strain variation for B1M1-59 at 68% maximum stress with number of cycles
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-24 Stress-strain curve and [b] total longitudinal strain variation for B1M1-61 at 68% maximum stress with number of cycles
Figure 4-25 Stress-strain curve and [b] total longitudinal strain variation for B1M1-62 at 68% maximum stress with number of cycles
Figure 4-26 Stress-strain curve and [b] total longitudinal strain variation for B1M1-63 at 68% maximum stress with number of cycles
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-27 Stress-strain curve and [b] total longitudinal strain variation for B1M1-64 at 68% maximum stress with number of cycles
Figure 4-28 Stress-strain curve and [b] total longitudinal strain variation for B1M1-74 at 63% maximum stress with number of cycles.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-29 Stress-strain curve and [b] total longitudinal strain variation for B1M1-80 at 63% maximum stress with number of cycles
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-30 Stress-strain curve and [b] total longitudinal strain variation for B1M1-82 at 63% maximum stress with number of cycles
Figure 4-31 Stress-strain curve and [b] total longitudinal strain variation for B1M1- 83 at 63% maximum stress with number of cycles
Figure 4-32 Stress-strain curve and [b] total longitudinal strain variation for B1M1-86 at 63% maximum stress with number of cycles
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 4-33 Stress-strain curve and [b] total longitudinal strain variation for B1M1-88 at 63% maximum stress with number of cycles
To allow comparison of strains recorded during pilot fatigue tests and quasi static loading (see section 4.3.1), strains during quasi-static tests at 1.2 N/mm$^2$ were evaluated (maximum applied stress quasi statically for the pilot tests was 1.2 N/mm$^2$ and 1.1 N/mm$^2$ for 68% and 63% maximum stress levels). Recorded strain during quasi-static tests varies from -0.002 mm/mm to -0.006 mm/mm (mean -0.004 mm/mm; standard deviation 0.002 mm/mm). Strain recorded after $N_f$ cycles of loading at 68% maximum stress level varies from -0.005 mm/mm to -0.016 mm/mm (mean -0.011mm/mm; standard deviation 0.005mm/mm). The respective variation for prisms that have been subjected to $N_f$ cycles of loading at 63% maximum stress level is from -0.017 mm/mm to -0.025 mm/mm (mean -0.020mm/mm; standard deviation 0.003mm/mm). Cyclic loading is, therefore, resulting in notably higher sustained strains and strains increase as the maximum applied stress decreases, probably due to the effect of creep (Figure 4-34).

![Strain at 63% and 68% stress levels together with strain during quasi-static tests (n=18)](image)

**Figure 4-34** Strain at 63% and 68% stress levels together with strain during quasi-static tests (n=18)
4.4 Discussion

The experimental results presented in chapter 4 describe the response of brick-masonry prisms under fatigue compressive loading. Previous research on the topic (section 2.4.2) has mainly focused on the maximum number of loading cycles to fatigue failure and only limited data is available on the deformation evolution during fatigue loading for brick masonry.

The failure modes observed during high-cycle fatigue loading within the current work are similar to failure modes observed by other researchers. Abrams et al. (1985) stated that although masonry fails at a lower maximum stress level during fatigue loading compared to the quasi-static loading, the failure modes are similar (vertical cracks forming through the bricks and mortar joints). Vertical splitting of the bricks and mortar accompanied by smooth patches on the broken surface of the bricks was also observed by Clark (1994). Ronca et al. (2004) were more specific by stating that no splitting cracks formed on the front side of the specimen. In contrast, vertical cracks were visible on the minor sides of the specimens and propagated with increased loading cycles until failure. Smaller cracks were also observed at the front and back of the specimens.

Test results indicate that the maximum stress level has significant impact on the fatigue life of masonry. Lower maximum stress levels lead to increased fatigue life as has been found by Tomor et al. (2013). However, Abrams et al. (1985) concluded that the maximum stress applied is not a reliable indicator of the fatigue life and that the amplitude of the cyclic load is more significant. Ronca et al. (2004) also identified the importance of the alternating stress value, while Roberts et al. (2006) suggested that the fatigue strength of masonry depends on the induced stress range but also on the mean or maximum induced stress and
the compressive strength of masonry. During the current test series determining the influence of the compressive strength, the stress range and the mean stress level was not possible, as the tests focussed on one specific masonry type (low-strength masonry) and the minimum stress level (10%) was the same for all tests.

Abrams et al. (1985) published results on the changes of the material properties during cyclic loading. Results indicated that that the accumulation of deformation is faster for prisms built with a stronger type mortar than for those with lower-strength mortars. The shape of the strain evolution curves are similar with the ones obtained from the current experimental data. Abrams et al. (1985), however, performed low-cycle fatigue tests (specimens that did not fail before 180 cycles were subsequently loaded quasi-statically to failure) and did not record the complete strain-loading cycles curves. No conclusions could, therefore, be drawn on the effect of the maximum stress level or the overall loading cycles on the strain accumulation. Ronca et al. (2004) tested masonry prisms under cyclic compression loading up to 11,600 cycles at different load frequencies (1 Hz, 5 Hz and 10 Hz) and presented the strain evolution curves. Based on the test results, the authors concluded that the strain rate is more sensitive to the mechanical properties of the material than to the loading frequency. The effect of frequency was not investigated in the current work. Carpinteri et al. (2014) presented strain evolution curves for masonry and identified three stages during fatigue deterioration. Duration and configuration of each stage agrees with the ones observed during the current research. Carpinteri et al. (2014), however, observed Stage II to last until 80%-90% of the fatigue life, while in the current tests stage II lasted until 76%-94% of the fatigue life. This difference is not significant and could be due to the different loading frequency (1.3 Hz) or to the small number of specimens tested by the authors (2-3 tests for each specimen type).
Even though this is the first time data have been presented on the evolution of stress-strain curves for masonry under high-cycle fatigue, the evolution of load-displacement curves for masonry under low-cycle fatigue was published by Abrams et al. (1985). The curve indicated that residual deformations accumulated for each cycle and that deformations were larger for the first cycles, subsequently decreased with each successive cycle and increased again during the last loading cycles. There is also a wealth of information available on the stress-strain evolution for concrete under fatigue loading that can provide a useful starting point for the analysis. Holmes (1982) considered changes introduced in the stress-strain curves for concrete with increased loading cycles. Curves are changing from concave (with respect to the strain axis) to a straight line and further to convex. Similar findings were presented by Crumley and Kennedy (1977), who also stated that the residual strain increases with increased loading cycles, but not at a constant rate. The stress-strain curves presented in the current chapter exhibit similar changes as described by Crumley and Kennedy (1977) during inducing loading cycles. The initially concave shape of the curves is not, however, always apparent.

Tomor et al. (2013) presented Acoustic Emission recordings of fatigue tests under compression on masonry prisms and identified three stages during fatigue deterioration. While acoustic emission amplitude data suggested three somewhat different stages form the current test results (stage I 0-32%, stage II 32-67%, stage III 67-100% of the fatigue life), average acoustic emission energy recordings showed very similar limits of different fatigue stages (stage I 0-10%, stage II 10-80%, stage III 80-100%) (Error! Reference source not found.).
4.5 Chapter summary

In chapter four, the data and experimental test results of clay bricks, mortar cubes and brick masonry prisms were presented for compressive quasi-static and long-term fatigue tests. The compressive strength, the Young’s modulus and the stress-strain curves for each material were evaluated based on the data of the quasi-static compressive tests. Failure mechanisms for masonry were also identified based on photographic evidence.

To identify the correlation between the maximum stress ($S_{\text{max}}$) and the loading cycles to failure ($N_f$) test data for prisms under long-term fatigue compression were used to plot S-N graphs. Strain evolution curves were also presented and different stages of fatigue identified. Failure mechanisms were identified and compared to quasi-static tests. Pilot test data were discussed to identify changes in the stress-strain relationship and in the change in total longitudinal strain with the loading cycles. Finally, findings were discussed and compared with relevant research findings in literature. Research on both masonry and concrete were considered.
CHAPTER FIVE

5. ANALYSIS

5.1 Introduction

In chapter five, the experimental data presented previously will be post-processed to develop mathematical models to characterise the behaviour of masonry under compressive fatigue loading. Mathematical models for concrete found in the literature are adopted to identify the stress - number of cycles - probability (S-N-P) curves for masonry. Mathematical models are also developed based on the experimental data for the evolution of total, elastic and plastic strains and Young’s modulus during fatigue deterioration for masonry prisms. A practical example is provided to demonstrate the application of the developed models. The analysis is followed by a brief discussion and current research findings are set within the wider frame of research on masonry bridges under fatigue loading.

5.2 S-N curves for masonry

Behaviour of a material under long-term fatigue loading is usually expressed through S-N curves or Wöhler curves. These are semi-logarithmic graphs of the induced stress (S) against the number of loading cycles to failure (Nf) on the logarithmic scale. Representative S-N curves and fatigue equations were produced through statistical analysis of the experimental test data for masonry. The analysis that follows is based on the experimental data presented in both section 4.3.2 and 4.3.3.
5.2.1 Available S-N models

Fatigue equations have been proposed to describe the S-N relationship for masonry in the past by several researchers (Roberts et al., 2006; Casas, 2009; Casas, 2011; Tomor & Verstrynge, 2013), as discussed in section 2.4.2. In 2009, Roberts et al. performed a number of quasi-static and high-cyclic fatigue tests on three different types of masonry test specimens under dry, wet and submerged conditions and proposed a lower bound fatigue strength curve (Equation 5-1).

\[
F(S) = \left(\frac{\Delta \sigma}{f_c} \sigma_{\text{max}}\right)^{0.5} = 0.7 - 0.05 \log N_f
\]

Eq. 5-1

Where \(f_c\) is the quasi-static compressive strength, \(\sigma_{\text{max}}\) is the maximum induced stress, \(\Delta \sigma\) is the induced stress range and \(N_f\) is the number of cycles until fatigue failure occurs.

In Figure 5-1 the fatigue test data collected during the current research and presented in Chapter 4 are plotted and coupled with the lower bound fatigue model proposed by Roberts et al. (2006). Although the model can be considered a lower bound of the fatigue life, it does not comprise a safe solution and does not follow the logarithmic relationship between stress level and fatigue life.
Casas (2009; 2011), based on a Weibull distribution, proposed a model for the fatigue capacity for brick masonry. The model was calibrated using the experimental data from Roberts et al. (2006) and a statistical regression analysis was performed on the fatigue data at every stress level to verify the efficacy of the Weibull distribution for the fatigue phenomenon. Using the results of the regression analysis and an exponential fatigue equation, a probability-based fatigue model was provided for brick masonry for various confidence levels (Equation 5-2).

\[ S_{\text{max}} = A * N_f^{-B(1-R)} \]  
Eq. 5-2

Where \( S_{\text{max}} \) is the ratio of the maximum applied stress over the quasi-static compressive strength, \( N_f \) is the numbers of cycles to fatigue failure, \( R \) is the ratio of the minimum over
the maximum applied stress levels and A, B are coefficients for the survival function. The coefficients A and B, calculated by Casas (2009) are given in Table 5-1 for various probabilities of survival. Equation 5-3 is the proposed fatigue equation for a survival probability of 95% for masonry under any conditions (dry, wet or submerged). In this equation instead of using the value for B corresponding to PS=0.95 (Table 5-1) Casas used the mean value of the coefficient B for PS > 0.6.

Table 5-1 Parameters of the fatigue equation proposed by Casas (2009) depending on the required confidence level

<table>
<thead>
<tr>
<th>Probability of survival PS</th>
<th>Coefficient A</th>
<th>Coefficient B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>1.1060</td>
<td>0.0998</td>
</tr>
<tr>
<td>0.90</td>
<td>1.3030</td>
<td>0.1109</td>
</tr>
<tr>
<td>0.80</td>
<td>1.4580</td>
<td>0.1095</td>
</tr>
<tr>
<td>0.70</td>
<td>1.4940</td>
<td>0.1023</td>
</tr>
<tr>
<td>0.60</td>
<td>1.4870</td>
<td>0.0945</td>
</tr>
<tr>
<td>0.50</td>
<td>1.4640</td>
<td>0.0874</td>
</tr>
</tbody>
</table>

\[ S_{\text{max}} = 1.106N^{-0.1034(1-R)}, S_{\text{max}} > 0.5 \quad \text{Eq. 5-3} \]

Figure 5-2 presents the current experimental data, as well as the proposed model from Casas (2009), for survival probability of 95% and 50%. For the 50% probability of survival curve coefficients A and B from table 5-1 were used (A = 1.4640 and B = 0.0874). The curve corresponding to the 50% probability of survival is intended to represent the mean values.
of the data at each stress level, while the 95% probability of survival curve to represent a lower bound. Both curves, however, do not seem to approximate the experimental data. Updated values for the coefficients A and B, based on the new experimental data are probably required.

![Graph showing fatigue test data and model](image)

**Figure 5-2** Fatigue test data together with the fatigue model proposed by Casas (2009) for probability of survival 95% and 50%.

Tomor and Verstrynge (2013) presented a joint fatigue-creep deterioration model for masonry based on two independent laboratory experimental test series. To incorporate both phenomena in one model, the creep test data were expressed by a mean stress with zero amplitude and one second of creep loading equated to two cycles under fatigue loading at 2 Hz frequency. Quasi-static test data were also included in the S-N curves expressed as failure at one cycle with a ratio of the maximum applied stress over the quasi-static compressive strength equal to one.
For relating the experimental results into a common mathematical expression, Tomor and Verstrynge (2013) adapted the fatigue model proposed by Casas (2009; 2011) and introduced a correction factor C (Equation 5-4).

\[ S_{\text{max}} = A \cdot N_f^{B(1-C\cdot R)} \]  
Eq. 5-4

Where \( S_{\text{max}} \) is the ratio of the maximum applied stress over the quasi-static compressive strength, \( N_f \) is the numbers of fatigue cycles to failure, R is the ratio of the minimum over the maximum applied stresses, parameter A is set to one, parameter B is set to 0.04 and C is the correction factor. Tomor and Verstrynge (2013) used a value of 0.62 for the correction factor C based on their experimental data.

In Figure 5-3 the current experimental fatigue data are collated with the joint fatigue-creep model proposed by Tomor and Verstrynge (2013). Parameters A and B were set to one and 0.04 respectively as suggested by the authors. For the correlation factor C values between -1.5 and 0.62 were selected to identify the best fit curve. The model is designed to represent the mean value of the fatigue life at each stress level (not a lower bound), and for C = -1.5 it seems to give a good approximation especially for stress levels below 70%. Experimental data are required, however, to identify the value of C for different types of masonry or set a range for it.
5.2.2 Proposed S-N-P model

Instead of presenting the fatigue test data for stress - number of cycles (S-N), it may be more conveniently presented in a three dimensional format using stress - number of cycles - probability of failure or probability of survival (S-N-P) due to its statistical nature. The S-N-P relationship has the ability to indicate curves for the lower bound, upper bound and the mean of the data points.

The probability of failure is calculated by ranking the specimens tested at each stress level in ascending order for the number of loading cycles to failure and dividing the rank ‘m’ of each specimen by (n+1), where ‘n’ is the total number of specimens tested at any specific stress level (Ross, 2009). By calculating the probability of failure (Pf) by dividing by (n+1) instead of ‘n’ helps to avoid Pf to be equal to 1 for the specimen that failed at the maximum number of cycles (Nf) and to account for even larger values of Nf (McCall, 1958; Zhao et al.,
Experimental and analytical investigations of brick masonry under compressive fatigue loading

The calculated values of probabilities of failure are given in Tables 5-2, 5-3 and 5-4 for different maximum stress levels.

Table 5-2 Number of cycles to failure and probability of failure at 55% and 60% maximum stress levels

<table>
<thead>
<tr>
<th>Specimen rank, m</th>
<th>P_f</th>
<th>N_f</th>
<th>log(N_f)</th>
<th>Specimen rank, m</th>
<th>P_f</th>
<th>N_f</th>
<th>log(N_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>2,200</td>
<td>3.34</td>
<td>1</td>
<td>0.13</td>
<td>25,342</td>
<td>4.40</td>
</tr>
<tr>
<td>2</td>
<td>0.22</td>
<td>4,864</td>
<td>3.69</td>
<td>2</td>
<td>0.25</td>
<td>122,762</td>
<td>5.09</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>56,562</td>
<td>4.75</td>
<td>3</td>
<td>0.38</td>
<td>796,744</td>
<td>5.90</td>
</tr>
<tr>
<td>4</td>
<td>0.44</td>
<td>412,774</td>
<td>5.62</td>
<td>4</td>
<td>0.50</td>
<td>986,325</td>
<td>5.99</td>
</tr>
<tr>
<td>5</td>
<td>0.56</td>
<td>1,088,560</td>
<td>6.04</td>
<td>5</td>
<td>0.63</td>
<td>1,268,627</td>
<td>6.10</td>
</tr>
<tr>
<td>6</td>
<td>0.67</td>
<td>1,672,237</td>
<td>6.22</td>
<td>6</td>
<td>0.75</td>
<td>2,646,302</td>
<td>6.42</td>
</tr>
<tr>
<td>7</td>
<td>0.78</td>
<td>1,724,587</td>
<td>6.24</td>
<td>7</td>
<td>0.88</td>
<td>3,528,118</td>
<td>6.55</td>
</tr>
<tr>
<td>8</td>
<td>0.89</td>
<td>10,225,676</td>
<td>7.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-3 Number of cycles to failure and probability of failure at 63% and 68% maximum stress levels

<table>
<thead>
<tr>
<th>Specimen rank, m</th>
<th>P_f</th>
<th>N_f</th>
<th>log(N_f)</th>
<th>Specimen rank, m</th>
<th>P_f</th>
<th>N_f</th>
<th>log(N_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06</td>
<td>54</td>
<td>1.7</td>
<td>1</td>
<td>0.05</td>
<td>34</td>
<td>1.53</td>
</tr>
<tr>
<td>2</td>
<td>0.13</td>
<td>256</td>
<td>2.41</td>
<td>2</td>
<td>0.09</td>
<td>134</td>
<td>2.13</td>
</tr>
<tr>
<td>3</td>
<td>0.19</td>
<td>266</td>
<td>2.43</td>
<td>3</td>
<td>0.14</td>
<td>212</td>
<td>2.33</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>269</td>
<td>2.43</td>
<td>4</td>
<td>0.18</td>
<td>1,100</td>
<td>3.04</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>543</td>
<td>2.74</td>
<td>5</td>
<td>0.23</td>
<td>1,800</td>
<td>3.26</td>
</tr>
<tr>
<td>6</td>
<td>0.38</td>
<td>718</td>
<td>2.86</td>
<td>6</td>
<td>0.27</td>
<td>3,000</td>
<td>3.48</td>
</tr>
<tr>
<td>7</td>
<td>0.44</td>
<td>881</td>
<td>2.95</td>
<td>7</td>
<td>0.32</td>
<td>3,541</td>
<td>3.55</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>1,104</td>
<td>3.04</td>
<td>8</td>
<td>0.36</td>
<td>3,600</td>
<td>3.56</td>
</tr>
<tr>
<td>9</td>
<td>0.56</td>
<td>2,515</td>
<td>3.40</td>
<td>9</td>
<td>0.41</td>
<td>5,994</td>
<td>3.78</td>
</tr>
<tr>
<td>10</td>
<td>0.63</td>
<td>3,355</td>
<td>3.53</td>
<td>10</td>
<td>0.45</td>
<td>6,737</td>
<td>3.83</td>
</tr>
<tr>
<td>11</td>
<td>0.69</td>
<td>4,809</td>
<td>3.68</td>
<td>11</td>
<td>0.50</td>
<td>11,754</td>
<td>4.07</td>
</tr>
<tr>
<td>12</td>
<td>0.75</td>
<td>11,038</td>
<td>4.04</td>
<td>12</td>
<td>0.55</td>
<td>13,000</td>
<td>4.11</td>
</tr>
<tr>
<td>13</td>
<td>0.81</td>
<td>19,203</td>
<td>4.28</td>
<td>13</td>
<td>0.59</td>
<td>17,350</td>
<td>4.24</td>
</tr>
<tr>
<td>14</td>
<td>0.88</td>
<td>34,728</td>
<td>4.54</td>
<td>14</td>
<td>0.64</td>
<td>18,276</td>
<td>4.26</td>
</tr>
<tr>
<td>15</td>
<td>0.94</td>
<td>59,921</td>
<td>4.78</td>
<td>15</td>
<td>0.68</td>
<td>18,651</td>
<td>4.27</td>
</tr>
<tr>
<td>16</td>
<td>0.73</td>
<td>31,000</td>
<td>4.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.77</td>
<td>33,752</td>
<td>4.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.82</td>
<td>37,938</td>
<td>4.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.86</td>
<td>69,537</td>
<td>4.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.91</td>
<td>71,342</td>
<td>4.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.95</td>
<td>250,000</td>
<td>5.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Several researchers have adopted the logarithmic normal distribution for fatigue life at constant amplitude stress for metals and concrete (Sinclair & Dolan, 1953; McCall, 1958; Stagg, 1970; Forrest, 1970; Zhao et al., 2015). Based on the logarithmic normal distribution, a graphical analysis similar to the one developed by McCall (1958) and Zhao et al. (2015) was performed to identify the S-N-P (stress-number of cycles-probability) curves for masonry. The probability density function (PDF) of the logarithmic normal distribution is given by Equation 5-5 (Zhao et al., 2015).

\[
f(N) = \left(\frac{1}{\sigma \sqrt{2\pi}}\right) \exp \left[-\frac{(\log N_f - \mu)^2}{2\sigma^2}\right] \tag{Eq. 5-5}
\]

Where \(\mu\) is the mean and \(\sigma\) is the standard deviation of \(\log N_f\) (listed in Table 5-5 for different stress levels). The cumulative density function can be obtained by integrating the probability density function (Equation 5-6).
Experimental and analytical investigations of brick masonry under compressive fatigue loading

\[ P(X \leq N_f) = \int_{-\infty}^{\log N_f} f(x) \, dx \]  

Eq. 5-6

Table 5-5 Mean and standard deviation of log\(N_f\) for different fatigue stress levels

<table>
<thead>
<tr>
<th>Stress level %</th>
<th>55</th>
<th>60</th>
<th>63</th>
<th>68</th>
<th>73</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.36</td>
<td>5.78</td>
<td>3.26</td>
<td>3.82</td>
<td>2.00</td>
<td>3.42</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.31</td>
<td>0.77</td>
<td>0.88</td>
<td>0.96</td>
<td>0.53</td>
<td>0.61</td>
</tr>
<tr>
<td>Number of specimens</td>
<td>8</td>
<td>7</td>
<td>15</td>
<td>21</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

In Figure 5-4 the calculated probabilities of failure at every stress level are plotted against the number of loading cycles to failure (\(N_f\)) in a semi-logarithmic scale together with the cumulative density function curves. The CDF curves were extrapolated to cover the whole probability range. The curves provide a good approximation of the fatigue test data indicating that the logarithmic normal distribution can describe the probability of failure.

Figure 5-4 Variation of failure probability with the loading cycles for different stress levels.
The fatigue lives corresponding to different probabilities of failure can be calculated from the N-P plot in Figure 5-4 and a family of stress level – number of cycles - probability of failure (S-N-P) curves can be generated. Figure 5-5 presents the curves corresponding to 0.05, 0.1, 0.5, 0.9 and 0.95 probabilities of failure calculated based on a power law best fit. The S-N curves are given by Equation 5-7.

\[ S_{\text{max}} = A \times N_f^B \]  
Eq. 5-7

Where A and B are parameters depending on the probability of failure (Table 5-6).

**Table 5-6 Parameters A and B for different probabilities of failure**

<table>
<thead>
<tr>
<th>Probability of Failure (Pf)</th>
<th>0.05</th>
<th>0.10</th>
<th>0.50</th>
<th>0.90</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.779</td>
<td>0.802</td>
<td>0.868</td>
<td>0.905</td>
<td>0.925</td>
</tr>
<tr>
<td>B</td>
<td>-0.028</td>
<td>-0.030</td>
<td>-0.030</td>
<td>-0.028</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

**Figure 5-5** Experimental data and predicted S-N curves for different probabilities of failure.
The curve corresponding to 0.50 probability of failure provides a good approximation of the mean test data. The curves corresponding to 0.05 and 0.10 probabilities of failure do not, however, represent lower bounds. This could be due to the fact that only a few specimens were tested at 80% maximum stress and results indicated greater fatigue lives than for 73% stress level. Additionally, extrapolation of the distributions to low probabilities resulted in intersection of distribution curves. This intersection produced the anomaly that below a certain probability, specimens tested at lower stress levels have shorter fatigue life. More test data are required for high stress levels so that this method may provide better results for lower bound S-N curves.

McCall (1958) used a mathematical model to describe the S-N-P relationship of fatigue data for plain concrete under reverse bending loading (Equation 5-8). The S-N-P relationship proposed by MacCall (1958) was adopted by a number of researchers (Ople & Hulsbos, 1966; Holmen, 1982; Singh et al., 2005; Singh & Kaushik, 2006; Singh et al., 2008; Graeff et al., 2012) for describing the behaviour of concrete under fatigue loading.

\[
L = 10^{-a S_{\text{max}}^b \left(\log N_f\right)^c}
\]

Eq. 5-8

Where a, b and c are experimental constants, \(S_{\text{max}}\) is the ratio of the maximum applied stress over the quasi-static compressive strength, \(N_f\) is the numbers of cycles until the occurrence of fatigue failure and \(L\) is the probability of survival. The probability of survival \(L\) is equal to 1-\(P_f\) (\(P_f\) is the probability of failure) and is used instead of the probability of failure in order to simplify the equation. In Equation 5-8, the following limits have been considered:

\[
L = 1 \text{ for } N_f = 1
\]
Experimental and analytical investigations of brick masonry under compressive fatigue loading

$L \to 0$ for $N_f \to \infty$

$L = 1$ for $S_{\text{max}} = 0$

$L \to 0$ for $S_{\text{max}} \to 1$

To evaluate the efficacy of the model for masonry, parameters $a$, $b$ and $c$ have to be calculated. The model of Equation 5-8 needs to be modified to account for different stress ranges as well as for the maximum stress level. The term $S_{\text{max}} \Delta S$ will be used for the analysis instead of $S_{\text{max}}$ to consider different minimum stress levels. Equation 5-8, therefore, is transformed to Equation 5-9.

$$L = 10^{-a(S_{\text{max}} \Delta S)^b (\log N_f)^c}$$

Eq. 5-9

First, the logarithms of the logarithms of each side of Equation 5-9 are found to transform the equation to a linear format (Equation 5-10).

$$\log(-\log L) = \log a + b \log(S_{\text{max}} \Delta S) + c \log(\log N_f)$$

Eq. 5-10

By substituting $\log(-\log L)$ by $Y$, $\log a$ by $A$, $\log(S_{\text{max}} \Delta S)$ by $X$ and $\log(\log N_f)$ by $Z$ a linear form is obtained (Equations 5-11), that can be rearranged in the form of Equation 5-12:

$$Y = A + bX + cZ$$

Eq. 5-11

or

$$Z = A' + B'X + C'Y$$

Eq. 5-12

Where $A' = -\frac{A}{c}$, $B' = -\frac{b}{c}$ and $C' = \frac{1}{c}$
In order to work with the variables measured from the sample, Equation 5-13 can be derived from Equation 5-12.

$$\sum Z = \sum A' + B' \sum X + C' \sum Y$$

$$\frac{1}{n} \sum Z = A' + B' \frac{1}{n} \sum X + C' \frac{1}{n} \sum Y$$

$$\bar{Z} = A' + B'\bar{X} + C'\bar{Y} \quad \text{Eq. 5-13}$$

If Equation 5-13 is subtracted from Equation 5-12, the following equation (Equation 5-14) is obtained:

$$Z - \bar{Z} = B'(X - \bar{X}) + C'(Y - \bar{Y})$$

or

$$z = b'x + c'y \quad \text{Eq. 5-14}$$

In Equation 5.14 $z = Z - \bar{Z}$, $x = X - \bar{X}$ and $y = Y - \bar{Y}$.

Using least square normal equations, the following expressions are obtained (Equations 5-15 and 5-16):

$$b' \sum x^2 + c' \sum xy = \sum xz \quad \text{Eq. 5-15}$$

$$b' \sum xy + c' \sum y^2 = \sum yz \quad \text{Eq. 5-16}$$

Analysing the experimental fatigue data based on this set of equations, the following statistical terms were calculated.
\[ \sum x^2 = 0.5533 \quad \sum xy = 0.0016 \quad \bar{x} = -0.4404 \]
\[ \sum y^2 = 11.5954 \quad \sum yz = 3.0258 \quad \bar{y} = -0.5803 \]
\[ \sum z^2 = 2.0887 \quad \sum xz = -0.5664 \quad \bar{z} = 0.5471 \]

Substitution of these statistical terms in Equations 5-15 and 5-16 allows the calculation of the parameters \( b' \) and \( c' \). Equation 5-14, using the calculated \( b' \) and \( c' \) parameters, becomes:

\[
z = -1.0243x + 0.2601y
\]

Parameter \( A' \) can now be calculated by substitution of \( B' \) and \( C' \) as well as \( \bar{X}, \bar{Y} \) and \( \bar{Z} \) in Equation 5-13. Equation 5-12 is expressed as:

\[
Z = 0.2474 - 1.0243X + 0.2609
\]

Finally, after calculation of all the required parameters (\( a, b \) and \( c \)), Equation 5-9 can be rewritten for masonry under compressive fatigue loading in the following form (Equation 5-17).

\[
L = 10^{-0.1127(S_{\text{max}}\Delta S)^{3.9252}(\log N_f)^{3.8322}} \quad \text{Eq. 5-17}
\]

Equation 5-17 can be used to evaluate the S-N curve for masonry under compressive cyclic loading for any preferred confidence level of survival. The P-S-N curves for 99%, 95%, 50%, 5% and 1% probability of survival are illustrated in Figure 5-6 with the experimental data.
The curves were plotted for 0.10 minimum stress level. The curve for 0.50 probability of survival gives a good approximation of the mean cycles to failure for each stress level, while the curves corresponding to 0.01 and 0.99 probability of survival form an upper and lower limit, respectively. Curves corresponding to 0.05 and 0.95 probability of survival can be used as less conservative upper and lower limits.

![Figure 5-6 S-N-P curves for masonry under fatigue loading at 2Hz with 10% minimum stress](image)

5.3 Changes in strain (ε-N) during fatigue deterioration

Based on the experimental data presented in Chapter 4 changes in the shape of the strain curves during fatigue deterioration (ε-N) is characterised by three distinct stages. Stage I up to ca. 10% of the fatigue life of parabolic shape, stage II up to 90% of the fatigue life with linear relationship between ε and N and stage III between 90% and 100% also of parabolic
shape. Next, a mathematical model will be developed to describe the evolution of changes in strain during fatigue deterioration.

5.3.1 Available $\epsilon$-$N$ models

There is no mathematical model available at present for describing the evolution of strain during the different stages of fatigue deterioration for masonry. Carpinteri et al. (2014) proposed an empirical relationship between the rate of variation of the vertical deformation $\vartheta \epsilon_v/\vartheta n$ during stage II and the number of cycles at fatigue failure (Equation 5-18).

$$N_f = a \left( \frac{\vartheta \epsilon_v}{\vartheta n} \right)^b$$  

Eq. 5-18

Where $\epsilon_v$ is the vertical deformation, $n$ the number of cycles and $a$ and $b$ are experimental coefficients, evaluated by performing a number of cycles on masonry specimens, up to the second stage. The model is, however, limited to stage II and experimental data are needed to predict parameters $a$ and $b$. A relation that can predict the strain at any time during the fatigue life of masonry without the need of experimental data is required. Models developed for concrete will be used, as a reference to develop a respective model for masonry.

Holmen (1982) carried out a series of fatigue tests under compression on concrete cubes and cylinders with constant amplitude and proposed the model for the total maximum strain variation for stage I (0% to 10% fatigue life) and stage II (10% to 80% fatigue life) as shown in Equations 5-19 and 5-20.
For $0 < \frac{N}{N_f} \leq 0.10$:

$$\varepsilon_{max} = \frac{\varepsilon_0}{S_{max}} \left[ S_{max} + 3.180(1.183 - S_{max}) \left( \frac{N}{N_f} \right)^{0.5} \right]$$

$$+ 0.413(0.147S_{min} + 0.853S_{max})^{1.184} ln(t + 1)$$

Eq. 5-19

For $0.10 < \frac{N}{N_f} \leq 0.80$:

$$\varepsilon_{max} = \frac{1.110\varepsilon_0}{S_{max}} \left[ 1 + 0.677 \left( \frac{N}{N_f} \right) \right]$$

$$+ 0.413(0.147S_{min} + 0.853S_{max})^{1.184} ln(t + 1)$$

Eq. 5-20

Where $S_{max}$ is the maximum stress level, $S_{min}$ the minimum stress level, $\varepsilon_{max}$ the maximum total strain, $\varepsilon_0$ is the maximum total strain after the first cycle of loading, $t$ is the duration of alternating load in hours, $N$ is the number of cycles and $N_f$ is the number of cycles at fatigue failure.

Sanchez (2008) analysed the proposed model by Holmen (1982) and highlighted that apart from not providing an expression for stage III, the slopes of the two equations (Equations 5.19 and 5.20) do not coincide at the intersection point ($\frac{N}{N_f}$=0.10), introducing gaps in the curvature.

Pfister et al. (2006) provided expressions on the rate of strain for the three stages of fatigue deterioration for concrete. These expressions are second order polynomial equations of the induced stress levels (maximum stress $S_{max}$ and minimum stress $S_{min}$). The rates of strain
at the first and third stages $\epsilon_{1,3}$ are expressed by the same equation (Equation 5-21) and the rate of strain during the second stage $\epsilon_2^*$ is expressed by a separate relationship (Equation 5-22).

\[
\begin{align*}
\epsilon_{1,3}^* &= -79.71s^2 + 46.53s - 1.76 \\
\epsilon_2^* &= -24.71s^2 + 19.19s - 2.63
\end{align*}
\]

where

\[s = (S_{\text{max}} - S_{\text{min}}) \left( \frac{S_{\text{max}} + S_{\text{min}}}{2} \right)\]

Zanuy (2008) calculated the deformation corresponding to $N/N_f = 0.10$ ($\epsilon_{1,2}$) and the strain rate in the second stage $\epsilon_2^*$ resulting from the model proposed by Holmen (1982) (from Equations 5.19 and 5.20) and proposed a new mathematical model. The deformation corresponding to $N/N_f = 0.10$, $\epsilon_{1,2}$ and the strain rate in the second stage $\epsilon_2^*$, used for the model are given by Equations 5-23 and 5-24.

\[
\begin{align*}
\frac{\epsilon_{\text{max}}}{\epsilon_0} \left( \frac{N}{N_f} = 0.10 \right) &= \epsilon_{1-2} = \frac{1.184}{S_{\text{max}}} \quad \text{Eq. 5-23} \\
\frac{d \left( \frac{\epsilon_{\text{max}}}{\epsilon_0} \right)}{d \left( \frac{N}{N_f} \right)} &= \epsilon_2^* = \frac{0.74037}{S_{\text{max}}} \quad \text{Eq. 5-24}
\end{align*}
\]

Zanuy (2008) proposed three expressions (Equations 5-25, 5-26 and 5-27) for the change in strain during stage I, stage II and stage III of fatigue life for concrete.

Model for strain evolution for $0 \leq \frac{N}{N_f} \leq 0.10$: 

132
\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left(\frac{N}{N_f}\right) = 1 + A \frac{N}{N_f} + B \left(\frac{N}{N_f}\right)^2
\]
Eq. 5-25

Where \( B = 100(1 - \varepsilon_{1-2}) + 10 \varepsilon_2, A = 20(\varepsilon_{1-2} - 1) - \varepsilon_2^* \)

Model for strain evolution for \( 0.10 \leq \frac{N}{N_f} \leq 0.90 \)

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left(\frac{N}{N_f}\right) = \varepsilon_{1-2} + \varepsilon_2^* \left(\frac{N}{N_f} - 0.1\right)
\]
Eq. 5-26

Model for strain evolution for \( 0.90 \leq \frac{N}{N_f} \)

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left(\frac{N}{N_f}\right) = \varepsilon_{1-2} + \varepsilon_2^* \left(\frac{N}{N_f} - 0.1\right) + C \left(\frac{N}{N_f} - 0.8\right)^2
\]
Eq. 5-27

Where \( C = 25 \left(\frac{\varepsilon_f}{\varepsilon_0} - \varepsilon_{1-2} - 0.9\varepsilon_2^*\right) \) and \( \varepsilon_f \) is the strain at failure.

### 5.3.2 Proposed \( \varepsilon \)-N model

Based on the same principles followed by researchers to describe the strain evolution over the fatigue life of concrete (Holmen, 1982; Pfister et al., 2006; Zanuy, 2008), three equations have been generated to characterise the three stages of the \( \varepsilon 

**Stage I (Second order parabola)**

Based on the experimental data presented in section 4.3.2 the mean duration of the three stages was calculated (Table 4-8). Stage I from the start of the test until 9.46% of the total loading cycles, stage II from 9.46% to 86.14% and stage III from 86.14% until failure. It was also observed from the experimental results that stage I and III can be approximated by a parabolic type equation and stage II by a linear equation. To simplify calculations, durations of stage I, II and III will be considered 0-10%, 10%-90% and 90%-100%, respectively.

A second order parabolic type equation was considered for stage I of fatigue (Equation 5-28).

\[ f(x) = ax^2 + bx + c \]  
Eq. 5-28

Substituting \( x = \frac{N}{N_f} \) and \( f(x) = \frac{\varepsilon_{max}}{\varepsilon_0} \) Equation 5.28 can be rewritten (Equation 5-29).

\[ \frac{\varepsilon_{max}}{\varepsilon_0} = a \left( \frac{N}{N_f} \right)^2 + b \frac{N}{N_f} + c, \quad \frac{N}{N_f} < 0.10 \]  
Eq. 5-29

The strain rate \( \varepsilon^* \) during stage I (Equation 5-30) is the tangent of the curve and can be evaluated from the first derivative of Equation 5-29.

\[ \varepsilon^* = \frac{\varepsilon_{max}}{\varepsilon_0} \frac{\partial N}{\partial N_f} = 2a \left( \frac{N}{N_f} \right) + b \]  
Eq. 5-30

In order to evaluate the parameters a, b and c in Equation 5-29 certain initial assumptions need to be considered.

- For \( N/N_f=0 \) strain is \( \varepsilon_{max}=\varepsilon_0 \) and therefore, \( \varepsilon_{max}/\varepsilon_0=1 \).
● Strain at the intersection point between stage I and stage II $\varepsilon_{1-2} = \frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left( \frac{N}{N_f} = 0.10 \right)$

...can be calculate from both equations for stage I and stage II and results should coincide.

● At the intersection point between stage I and stage II ($N/N_f=0.10$) the strain rate calculated from equation for stage I ($\varepsilon^*$) should coincide to the strain rate of stage II $\varepsilon_2^* = \frac{\partial \varepsilon_{\text{max}}}{\partial N_f} \left( \frac{N}{N_f} = 0.10 \right)$

Using the first assumption one can calculate parameter $c$ to be $c=1$. The second assumption ensures that there is no gap at the intersection points between the two stages and the third assumption assures continuity in terms of curvature.

To evaluate relationships of the maximum stress level with the strain at the end of stage I ($\varepsilon_{1-2}$) and the strain rate during stage II ($\varepsilon^*_2$ for $0.10 \leq N/N_f \leq 0.90$), the respective test data were plotted against maximum stress in Figure 5-7 and Figure 5-8. The best fit curves were identified as shown in Equations 5.31 and 5.32.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 5-7 Curve fitting for strain test data against maximum stress for the intersection point between stage I and stage II of the fatigue life

\[ \varepsilon_{1,2} = 4.256\left( \frac{S_{\text{max}}}{S_u} \right)^2 + 4.80\frac{S_{\text{max}}}{S_u} + 0.1369 \]

\[ r^2 = 0.607 \]

Figure 5-8 Curve fitting for the strain rate test data against maximum stress for stage II of the fatigue life

\[ \varepsilon'_{2} = 12.23\left( \frac{S_{\text{max}}}{S_u} \right)^2 - 15.58\frac{S_{\text{max}}}{S_u} + 0.081 \]

\[ r^2 = 0.734 \]
Experimental and analytical investigations of brick masonry under compressive fatigue loading

\[ \varepsilon_{1-2} = -4.256(S_{\text{max}})^2 + 4.80S_{\text{max}} + 0.1369 \quad \text{Eq. 5-31} \]

\[ \varepsilon'_{2} = 12.23(S_{\text{max}})^2 - 15.58S_{\text{max}} + 6.081 \quad \text{Eq. 5-32} \]

Substituting \( \varepsilon_{\text{max}}/\varepsilon_0 = 1 \) for \( N/N_f = 0 \), as well as, \( \varepsilon_{1-2} \) (from Equation 5-31) in Equation 5-29 and \( \varepsilon'_2 \) (from Equation 5-32) in Equation 5-30 for \( N/N_f = 0.1 \) parameters a,b and c were evaluated:

\[ a = 547.9S_{\text{max}}^2 - 635.8S_{\text{max}} + 147.12 \]

\[ b = -97.35S_{\text{max}}^2 + 111.58S_{\text{max}} - 23.343 \]

\[ c = 1 \]

Equation 5-29 can now be rewritten to describe the evolution of strain during the first stage of fatigue life as (Equation 5-33):

\[ \frac{\varepsilon_{\text{max}}}{\varepsilon_0} = (547.9S_{\text{max}}^2 - 635.8S_{\text{max}} + 147.12) \left( \frac{N}{N_f} \right)^2 \]

\[ - (97.35S_{\text{max}}^2 - 111.58S_{\text{max}} + 23.343) \frac{N}{N_f} + 1 \]

**Stage II (Linear)**

The equation for stage II of the fatigue life is of linear type (Equation 5-34).

\[ f(x) = ax + b \quad \text{Eq. 5-34} \]

Substituting \( x = N/N_f \) and \( f(x) = \frac{\varepsilon_{\text{max}}}{\varepsilon_0} \) Equation 5.34 can be rewritten (Equation 5-35).
The strain rate during stage II (Equation 5-36) can be evaluated from the first derivative of Equation 5-35.

\[
\varepsilon_2^* = \frac{\vartheta \varepsilon_{\text{max}}}{\varepsilon_0} = a
\]

Similar assumption as for stage I need to be considered to evaluate parameters a and b in Equation 5-35.

- Equations for stage I and stage II should provide the same result for strain for \(N/N_f = 0.10\) equal to \(\varepsilon_1^* = \frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left( \frac{N}{N_f} = 0.10 \right)\)

- The strain rate during stage II (0.10 < \(\frac{N}{N_f}\) < 0.90) must coincide to the strain rate at \(N/N_f = 0.10\) calculated for stage I \(\varepsilon_2^* = \frac{\vartheta \varepsilon_{\text{max}}}{\varepsilon_0} \left( \frac{N}{N_f} = 0.10 \right)\)

Substituting \(\varepsilon_1^*\) (from Equation 5-31) in Equation 5-35 for \(N/N_f = 0.1\) and \(\varepsilon_2^*\) (from Equation 5-32) in Equation 5-36 allows calculation of parameters a and b.

\[
a = \varepsilon_2^* = 12.23(S_{\text{max}})^2 - 15.58S_{\text{max}} + 6.081
\]

\[
b = -5.479S_{\text{max}}^2 + 6.358S_{\text{max}} - 0.4712
\]

The equation for the strain during stage II is, therefore, as per Equation 5-37.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} = \left[12.23S_{\text{max}}^2 - 15.58S_{\text{max}} + 6.081\right] \frac{N}{N_f} - 5.479S_{\text{max}}^2 + 6.358S_{\text{max}} - 0.4712 \tag{Eq. 5-37}
\]

Or

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} = \varepsilon^*_2 \frac{N}{N_f} + b, \quad 0.10 < \frac{N}{N_f} < 0.90 \tag{Eq. 5-38}
\]

**Stage III (Second order parabola)**

The \(\varepsilon\)-\(N\) equation for stage III of fatigue life is a second order parabolic type (Equation 5-39) as for stage I. The strain rate is again evaluated through derivation (Equation 5-40).

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} = a \left(\frac{N}{N_f}\right)^2 + b \frac{N}{N_f} + c, \quad 0.90 < \frac{N}{N_f} \tag{Eq. 5-39}
\]

\[
\varepsilon^* = \frac{\varepsilon_{\text{max}}}{\varepsilon_0} = 2a \left(\frac{N}{N_f}\right) + b \tag{Eq. 5-40}
\]

The initial assumptions made for the evaluation of parameters \(a\), \(b\) and \(c\) are:

- For \(\frac{N}{N_f} = 1.00\) (at failure) the strain is \(\varepsilon_f\)
- For \(\frac{N}{N_f} = 0.90\) the strain is \(\varepsilon_{2-3} = \frac{\varepsilon_{\text{max}}}{\varepsilon_0} \left(\frac{N}{N_f} = 0.90\right)\) and must coincide to the strain calculated using Equation 5.38 for stage II
- For \(\frac{N}{N_f} = 0.90\) the strain rate is \(\varepsilon^*_2\) as per Equation 5.32

139
The strain at the intersection between stage II and stage III $\varepsilon_{2-3}$ is calculated using Equation 5-39 and shown in Equation 5-41.

$$\varepsilon_{2-3} = 5.528\sigma_{\text{max}}^2 - 7.664\sigma_{\text{max}} + 5.002 \quad \text{Eq. 5-41}$$

To evaluate the relationship between the strain at failure $\varepsilon_f$ and the maximum stress level, test data were plotted against stress in Figure 5-9 and the best fit relationship was found as shown in Equation 5-42.

$$\varepsilon_f = 14.57\left(\frac{\sigma_{\text{max}}}{\sigma_u}\right)^2 - 24.11\frac{\sigma_{\text{max}}}{\sigma_u} + 12.88 \quad r^2 = 0.957$$

![Figure 5-9 Curve fitting for strain test data at failure against maximum stress](image)

Substituting $\varepsilon = \varepsilon_{2-3}$ and $\varepsilon^* = \varepsilon_{2}^*$ for $N/N_f=0.9$ in Equation 5-39 and 5-40 and $\varepsilon/\varepsilon_0 = \varepsilon_f$ for $N/N_f=1$ in Equation 5-39 a system of three equations with three variables is obtained. Solving the system of equations results in calculating parameters a, b and c.
\[ a = 781.9 S_{max}^2 - 1488.8 S_{max} + 727.02 \]
\[ b = -1395.19 S_{max}^2 + 2664.26 S_{max} - 1302.555 \]
\[ c = 627.86 S_{max}^2 - 1199.57 S_{max} + 588.415 \]

Equation 5-39 can now be rewritten (Equation 5-43) to predict strain during the stage III of fatigue life:

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} = \left[ 781.9 S_{max}^2 - 1488.8 S_{max} + 727.02 \right] \left( \frac{N}{N_f} \right)^2 \\
+ \left[ -1395.19 S_{max}^2 + 2664.26 S_{max} - 1302.555 \right] \frac{N}{N_f} \\
+ [627.86 S_{max}^2 - 1199.57 S_{max} + 588.415]
\]

Eq. 5-43

The recorded strain during the fatigue tests is shown together with the prediction model in Figure 5-10 and Figure 5-11 for the various maximum stress levels (55%, 60%, 68%, 80%). Good correlation is found between the proposed mathematical model and test data. The curve for stage I and the linear equation for stage II are generally good representations of the test data for all stress levels. The curvature for stage III provides a good representation for stress levels 68% and 80%, although it is slightly steeper than the test data for 55% and 60% stress. The start and end of the curves for stage III, however, corresponds to the mean strain at the respective points and can be considered as acceptable.

The slopes of the curves coincide at the intersection points between the adjoining stages, without any gaps (values for \( \varepsilon/\varepsilon_0 \) coincide to the 10\(^{th} \) decimal in the mathematical model for \( N/N_f=0.1 \) and \( N/N_f=0.9 \)). Coincidence of inclinations and numerical values at intersection points shape a continuous differentiable function.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 5-10 Total strain and proposed mathematical model against number of cycles ratio for (a) 55%, (b) 60%
Figure 5-11 Total strain and proposed mathematical model against number of cycles ratio for (a) 68%, (b) 80%
5.3.3 Proposed model for elastic and plastic strain evolution

Total strain is a sum of the elastic $\varepsilon_{el}$ and plastic strain $\varepsilon_{pl}$ (Equation 5-44).

$$\varepsilon_{tot} = \varepsilon_{el} + \varepsilon_{pl}$$  \hspace{1cm} \text{Eq. 5-44}

Elastic strain is reversible and is an indication of the deformability and endurance of the material, while plastic strain is irreversible, stress and time dependant and is related to creep effects (Tomor & Verstrynge, 2013). The experimental data from the pilot tests presented in section 4.3.3 have been further processed to study the elastic end plastic components of the total strain during fatigue deterioration. The maximum stress levels studied during the pilot tests were 63%, 68% and 73%. There are no results, however, for 73% maximum stress level.

During the pilot tests, prisms were loaded cyclically for 1000 cycles, then unloaded and loaded again quasi-statically up to the mean stress level to follow another 1000 cycles of loading. The total, elastic and plastic strain were evaluated based on the recordings from LVDTs for the quasi-static loading part as shown in Figure 5-12.

![Figure 5-12 Schematic of stress - strain curve for quasi-static loading and elastic and plastic components of strain](image)
The total strain (Figure 5-13) follows the evolution law described earlier in section 5.3.2. The shape of the plastic strain evolution curve (Figure 5-14) is similar to that of total strain, however, the shape of the elastic strain evolution curve (Figure 5-15) does not follow the same trend and seems to grow more steadily with the number of cycles.

To quantify the elastic ($\varepsilon_{el}$) and plastic ($\varepsilon_{pl}$) components of strain with the total strain ($\varepsilon_{tot}$), the $\varepsilon_{pl}/\varepsilon_{tot}$ and $\varepsilon_{el}/\varepsilon_{tot}$ ratios are plotted against the number of loading cycles in Figure 5-16 and Figure 5-17 for stress levels 63% and 68% (test data presented in Section 4.3.3).
Figure 5-13 Changes in total strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress
Figure 5-14 Changes in plastic strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress
Figure 5-15 Changes in elastic strain against number of cycles ratio for (a) 63% and (b) 68% maximum stress
Figure 5-16 Strain against number of cycles ratio for (a) $\varepsilon_{pl}/\varepsilon_{tot}$ and (b) $\varepsilon_{el}/\varepsilon_{tot}$ for 63% maximum stress
Figure 5-17 Strain against number of cycles ratio for (a) $\varepsilon_{pl}/\varepsilon_{tot}$ and (b) $\varepsilon_{el}/\varepsilon_{tot}$ for 68% maximum stress
Following the same procedure as presented for the total longitudinal strain in Section 5.3.2, a prediction model for the plastic strain ratio \(\frac{\varepsilon_{pl}}{\varepsilon_{tot}}\) is identified with three stages as well. For stage I and III the equation is of second order parabolic type and for stage II a linear equation. The elastic strain ratio \(\frac{\varepsilon_{el}}{\varepsilon_{tot}}\) is calculated, subsequently, as \(\frac{\varepsilon_{el}}{\varepsilon_{tot}} = 1 - \frac{\varepsilon_{pl}}{\varepsilon_{tot}}\) according to Equation 5-44.

Data collected from the pilot tests are limited and only for 63% and 68% maximum stresses. It is, therefore, not possible to propose a model able to predict strains at any stress level. The model generated describes only the available data and consists a starting point for future research.

**Stage I (Second order parabola)**

A second order parabolic equation (Equation 5-45) can describe the \(\frac{\varepsilon_{pl}}{\varepsilon_{tot}}\) curve for \(0 < \frac{N}{N_f} < 0.1\).

\[
\frac{\varepsilon_{pl}}{\varepsilon_{tot}} = a \left(\frac{N}{N_f}\right)^2 + b \left(\frac{N}{N_f}\right) + c
\]

Eq. 5-45

Integration of Equation 5-45 gives the rate of \(\frac{\varepsilon_{pl}}{\varepsilon_{tot}}\) for stage I of fatigue (Equation 5-46).

\[
\varepsilon^* = \frac{\theta}{\theta} \frac{\varepsilon_{pl}}{\varepsilon_{tot}} = 2a \left(\frac{N}{N_f}\right) + b
\]

Eq. 5-46
Similar assumptions were made as in section 5.3.2 to evaluate the parameters \(a, b, c\) in Equation 5-45 and ensure continuity of the curve between adjoining stages.

- For \(N/N_f=0\) strain is \(\varepsilon_{pl}/\varepsilon_{tot}=0\).
- Equations for stage I and stage II should provide the same result for plastic strain for \(N/N_f=0.10\) equal to \(\varepsilon_{pl}^{1-2} = \varepsilon_{tot}\left(\frac{N}{N_f} = 0.10\right)\).
- For \(N/N_f=0.10\) the plastic strain rate calculated from equation for stage I should coincide to the strain rate of stage II \(\varepsilon_{pl,2}^* = \frac{\delta \varepsilon_{pl}}{\delta \varepsilon_{tot}}\left(\frac{N}{N_f} = 0.10\right)\).

Experimental data were plotted at \(N/N_f=0.1\) (Figure 5-18 and Figure 5-19) to identify the relationship of the plastic strain and the plastic strain rate with the maximum stress (Equations 5-47 and 5-48). Due to the limited test data (only two stress levels) identification of a more accurate fitting curve is not possible and a linear relationship based on least square equations was assumed.

![Figure 5-18 Curve fitting for plastic strain test data against maximum stress for the intersection point between stage I and stage II of the fatigue life](image)

\[\varepsilon_{pl}/\varepsilon_{tot} = -0.0291S_{max} + 2.2774 \quad R^2 = 1\]
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 5-19 Curve fitting for the plastic strain rate test data against maximum stress for stage II of the fatigue life

\[ \varepsilon_{pl}^{1-2} = -2.9133S_{max} + 2.2774 \]  
Eq. 5-47

\[ \varepsilon_{pl,2}^* = -1.7491S_{max} + 1.3113 \]  
Eq. 5-48

Substitution of Equations 5-47 and 5-48 in Equations 5-45 and 5-46 respectively for \( N/N_f = 0.1 \) and \( \varepsilon_{pl}/\varepsilon_{tot} = 0 \) for \( N/N_f = 0 \) allows calculation of parameters \( a, b \) and \( c \). Equation 5-45 is transformed into Equation 5-49.

\[ a = 273.839S_{max} - 214.627 \]

\[ b = -56.517S_{max} + 44.237 \]

\[ c = 0 \]

\[ \frac{\varepsilon_{pl}}{\varepsilon_{tot}} = \left(273.839S_{max} - 214.627\right)\left(\frac{N}{N_f}\right)^2 \]

\[ + \left(-56.517S_{max} + 44.237\right)\left(\frac{N}{N_f}\right) \]  
Eq. 5-49

153
Stage II (Linear)

For \(0.1 < \frac{N}{N_f} < 0.9\) a linear equation is characterising the evolution of plastic strain (Equation 5-50):

\[
\frac{\varepsilon_{pl}}{\varepsilon_{tot}} = a \left( \frac{N}{N_f} \right) + b \tag{Eq. 5-50}
\]

The plastic strain rate during stage II (Equation 5-51) can be evaluated from the first derivative of Equation 5-50.

\[
\dot{\varepsilon}_{pl,2} = \dot{\varepsilon_0} \frac{\varepsilon_{max}}{\dot{\varepsilon_0} \frac{N}{N_f}} = a \tag{Eq. 5-51}
\]

Using the second assumption from stage I and substituting Equations 5-47 and 5-48 in Equations 5-50 and 5-51 for \(N/N_i=0.1\) parameters \(a\) and \(b\) were calculated and the relationship for stage II is given by Equation 5-52.

\[
a = -1.749S_{max} + 1.311
\]

\[
b = -2.738S_{max} + 2.146
\]

\[
\frac{\varepsilon_{pl}}{\varepsilon_{tot}} = (-1.749S_{max} + 1.311) \left( \frac{N}{N_f} \right) + (-2.738S_{max} + 2.146) \tag{Eq. 5-52}
\]
Stage III (Second order parabola)

Finally for stage III of fatigue \((0.9 < \frac{N}{N_f} < 1.0)\) a second order parabolic equation was assumed to fit the experimental data (Equation 5-53). The plastic strain rate (Equation 5-54) is again evaluated through derivation of Equation 5-53.

\[
\frac{\varepsilon_{pl}}{\varepsilon_{tot}} = a \left( \frac{N}{N_f} \right)^2 + b \left( \frac{N}{N_f} \right) + c \tag{Eq. 5-53}
\]

\[
\varepsilon^* = -\frac{\varepsilon_{pl}}{\varepsilon_{tot}} \left( \frac{N}{N_f} \right) = 2a \left( \frac{N}{N_f} \right) + b \tag{Eq. 5-54}
\]

The initial assumptions made for the evaluation of parameters \(a\), \(b\) and \(c\) are:

- For \(\frac{N}{N_f} = 1.00\) (at failure) the strain is \(\varepsilon_{pl,f}\).
- for \(\frac{N}{N_f} = 0.90\) the plastic strain is \(\varepsilon_{pl}^{2-3} = \frac{\varepsilon_{pl}}{\varepsilon_{tot}} \left( \frac{N}{N_f} = 0.90 \right)\) and must coincide to the strain calculated using Equation 5-52 for stage II.
- For \(\frac{N}{N_f} = 0.90\) the plastic strain rate is \(\varepsilon_{pl,2}^*\) as per Equation 5-48.

The plastic strain at the intersection between stage II and stage III \(\varepsilon_{pl}^{2-3}\) is calculated by substituting \(N/N_f=0.9\) in Equation 5-52. The resulting expression is as per Equation 5-55.

\[
\varepsilon_{pl}^{2-3} = -4.3126S_{max}^2 + 3.3264S_{max} \tag{Eq. 5-55}
\]

To evaluate the relationship between the plastic strain at failure \(\varepsilon_{pl,f}\) and the maximum stress level, test data were plotted against stress in Figure 5-20 and a linear relationship was identified using least square expressions Equation 5-56.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 5-20 Curve fitting for plastic strain test data against maximum stress at failure

\[ \varepsilon_{pl,f} = -5.0046S_{\text{max}} + 3.8619 \]  
Eq. 5-56

Substitution of Equation 5-55 in Equation 5-53 for \( N/N_f = 0.9 \), Equation 5-51 in Equation 5-54 for \( N/N_f = 0.9 \) and Equation 5-56 in Equation 5-53 for \( N/N_f = 1 \) generates a system of three equations with three variables. Parameters \( a \), \( b \) and \( c \) were calculated after solving the system of equations and the relationship for plastic strain during stage III identified (Equation 5-57).

\[ a = -51.711S_{\text{max}} + 40.433 \]

\[ b = 91.331S_{\text{max}} - 71.468 \]

\[ c = -44.624S_{\text{max}} + 34.897 \]
\[
\frac{\varepsilon_{pl}}{\varepsilon_{tot}} = (-51.711S_{max} + 40.433)\left(\frac{N}{N_f}\right)^2 + (91.331S_{max} - 71.468)\left(\frac{N}{N_f}\right) + (-44.624S_{max} + 34.897) \quad \text{Eq. 5-57}
\]

Using Equations 5-49, 5-52 and 5-57 the plastic \((\varepsilon_{pl}/\varepsilon_{tot})\) and elastic \((\varepsilon_{el}/\varepsilon_{tot})\) strain ratios are plotted against the number of cycles ratios in Figure 5-21 and Figure 5-22 for 63% and 68% maximum stress, respectively. The model seems to be predicting the test data with good accuracy for both stress levels. It should, however, be noted, that the mathematical model was developed based on limited range of test data for prisms tested under 63% and 68% maximum stress levels. Therefore, the expressions for \(\varepsilon_{pl}/\varepsilon_{tot}\) and \(\varepsilon_{el}/\varepsilon_{tot}\) are only valid for the specific stress range. More experimental data are required to develop a set of equations to predict the evolution of plastic and elastic strains for masonry.
Figure 5-21 Test data and proposed mathematical model against number of cycles ratio for (a) plastic ($\epsilon_{pl}/\epsilon_{tot}$) and (b) elastic ($\epsilon_{el}/\epsilon_{tot}$) strain for 63% maximum stress.
Figure 5-22 Test data and proposed mathematical model against number of cycles ratio for (a) plastic ($\varepsilon_{pl}/\varepsilon_{tot}$) and (b) elastic ($\varepsilon_{el}/\varepsilon_{tot}$) strain for 68% maximum stress.
5.4 Changes in the Young’s modulus (E) during fatigue deterioration

5.4.1 Available models for the Young’s modulus

Several researchers have studied the Young’s modulus for concrete during fatigue deterioration and concluded that it decreases with increasing loading cycles (Crumley & Kennedy, 1977; Holmen, 1982; Cachim et al., 2002; Mu & Shah, 2005; Breitenbucher & Ibuk, 2006; Zanuy et al., 2011; Vicente et al., 2014). For masonry, the only relevant study was presented by Alshebani & Sinha (2001) who also found that the Young’s modulus decreases with increasing number of cycles. According to Alshebani & Sinha (2001) deterioration initiates at ca. 20% of the maximum load capacity, below which the stiffness is relatively constant.

5.4.2 Proposed model for the Young’s modulus

Based on the test data presented in section 4.3.2 and 4.3.3, the Young’s modulus was calculated as a secant modulus between the minimum and maximum applied stresses based on the readings from the load cell for specimens tested under fatigue loading (Figure 5-23). The evolution of Young’s modulus with the number of loading cycles is plotted for all prisms for 55%, 60%, 68% and 80% maximum stresses in Figure 5-24 and Figure 5-25. Changes in the Young’s modulus during fatigue deterioration has also been plotted for the experimental data from the pilot tests for 63% (Figure 5-26a) and 68% (Figure 5-26b) maximum stress level. Both the Young’s modulus and the cycles of loading are expressed as percentage of the initial Young’s modulus and total number of cycles.
Figure 5-23 Schematic representation of the way Young’s modulus was calculated as a secant modulus between the minimum and maximum stress.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Figure 5-24 Changes in Young’s modulus against number of cycles ratio for (a) 55% and (b) 60% maximum stress
Figure 5-25 Changes in Young’s modulus against number of cycles ratio for (a) 68% and (b) 80% maximum stress
Figure 5-26 Changes in Young’s modulus against number of cycles ratio for (a) 63% and (b) 68% maximum stress
The mean, maximum and minimum values for the residual over the initial Young’s modulus are presented in Figure 5-27. The residual Young’s modulus $E_f$ can decrease down to ca. 75% of its initial value $E_0$, however, the mean value of $E_f$ lies between 86% and 89% of $E_0$.

**Figure 5-27 Residual over initial Young’s modulus against the maximum stress**

The evolution of the Young’s modulus for masonry during fatigue deterioration (Figure 5-24 to Figure 5-26) does not appear to follow the same trend observed for the strain evolution. Instead of the three stages related to fatigue deterioration, only two stages may be observed. The Young’s modulus changes at a constant rate up to ca. 95% of the fatigue life and decreases suddenly before failure. For a number of prisms, the second stage cannot be observed and the Young’s modulus decreases at a constant rate throughout the fatigue life.

Taking into consideration that the Young’s modulus changes at a constant rate up to ca. 95% of the fatigue life, the hypothesis that a linear equation can describe the evolution of
stiffness during fatigue was made (Equation 5-58). The first derivative of this relationship gives the inclination of the line (Equation 5-59).

\[
\frac{E}{E_0} = a \frac{N}{N_f} + b
\]

Eq. 5-58

\[
\frac{\partial \frac{E}{E_0}}{\partial \frac{N}{N_f}} = a
\]

Eq. 5-59

Substitution of \(E/E_0=1\) for \(N/N_f=0\) in Equation 5-58 gives \(b=1\). To calculate parameter \(a\), the inclination of the line that is determined by the initial \((E_0)\) and the residual Young’s modulus at \(N/N_f=1\) \((E_f)\) was calculated for all the maximum stress levels studied. Mean test data are plotted against the maximum stress (Figure 5-28) and the best fit curve was identified (Equation 5-60).

![Figure 5-28 Slope defined by the initial and the residual Young’s Modulus at failure against the maximum stress (n=42)](image-url)
\[ \frac{E}{E_0} = a = -3.0181S_{max}^3 + 5.6894S_{max}^2 - 3.5118S_{max} + 0.6175 \text{ Eq. 5-60} \]

Substituting parameters \( a \) and \( b \) in Equation 5-58 the following relationship (Equation 5-61) is obtained for the normalised Young’s modulus ratio \( (E/E_0) \).

\[ \frac{E}{E_0} = 1 - (3.0181S_{max}^3 - 5.6894S_{max}^2 + 3.5118S_{max} - 0.6175)\left(\frac{N}{N_f}\right) \text{ Eq. 5-61} \]

The test data were plotted again together with the prediction model (Equation 5-61) for different stress levels (Figure 5-29 to Figure 5-31) to evaluate the suitability of the prediction model to describe the stiffness deterioration of masonry during fatigue. The prediction model does not follow the configuration of the curve precisely, as a linear relationship was initially assumed. It can, however, be used to provide a prediction of the mean \( E/E_0 \) during fatigue life. It is, however, only for 63\% maximum stress that the prediction model appears to underestimate the test data.
Figure 5-29 Changes in Young’s modulus against number of cycles ratio together with prediction model for (a) 55% and (b) 60% maximum stress
Figure 5-30 Changes in Young's modulus against number of cycles ratio together with prediction model for (a) 68% and (b) 80% maximum stress
Figure 5-31 Changes in Young’s modulus against number of cycles ratio together with prediction model for (a) 63% and (b) 68% maximum stress
5.5 Practical Example

The Cavone Bridge in Pisticci, Italy (Figure 5-32) was built before the Second World War and is composed of three main (24.1 m maximum span and 1.1 m minimum thickness) and four secondary (11.8 m maximum span length and 0.7 m thickness) brick masonry arches. The total length is 140 m and the width is 5.6 m. The external layer of the piers consists of regular stone blocks containing a core of cohesive backfill, while the live load is distributed from the deck to the arches through an incoherent backfill. The abutments and spandrel walls consist of regular stone blocks (Laterza et al., 2016).

Laterza et al. (2016) assessed the Cavone Bridge to evaluate the remaining service life based on the fatigue models proposed by Roberts et al. (2006) and Casas (2009). The findings of this research will be used to demonstrate how the models proposed in Sections 5.2.2, 5.3.2 and 5.4.2 can be used in practice.

Laterza et al. (2016) used the Fatigue Model 3 of Eurocode 1 (2002) and an axle load of 120 kN for assessing the fatigue life of the bridge (Figure 5-33). This fatigue model is appropriate for typical heavy traffic on European main roads or motorways. Traffic category 2 according to Eurocode 1 (2002) was selected (Table 5-7) by the authors. The number of vehicles per year and per lane for this category is $0.5 \times 10^6$. For similar flow of traffic over the previous years, for approximately 80 years of service, $4 \times 10^7$ vehicles have crossed the bridge. Assuming heavy vehicles of two axles, the structure has sustained $8 \times 10^7$ loading cycles. For a thorough assessment of the residual life, however, a more detailed study on the traffic changes in terms of number of vehicles and weight is required. The residual life can then be evaluated using Miner’s rule.
Figure 5-32 Layout of the Cavone Bridge
Figure 5-33 Representation of the axle positions for fatigue Model 3 according to Eurocode 1 (2002)

Table 5-7 Number of heavy vehicles expected per year and per slow lane according to Eurocode 1 (2002)

<table>
<thead>
<tr>
<th>Traffic categories</th>
<th>Nobs per year and per slow lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  Roads and motorways with 2 or more lanes per direction with high flow rates of lorries</td>
<td>2.0 x 10^6</td>
</tr>
<tr>
<td>2  Roads and motorways with medium flow rates of lorries</td>
<td>0.5 x 10^6</td>
</tr>
<tr>
<td>3  Main roads with low flow rates of lorries</td>
<td>0.125 x 10^6</td>
</tr>
<tr>
<td>4  Local roads with low flow rates of lorries</td>
<td>0.05 x 10^6</td>
</tr>
</tbody>
</table>

Laterza et al. (2016) adopted the fatigue assessment procedure for steel elements according to the Italian Code (NTC-08, 2009) and Eurocode 3 (2005), in the absence of a relative procedure for masonry elements. According to the Italian Code NTC-08 (2009), two different values for the masonry compressive strength associated to knowledge levels (KL1 and KL3 in Table 5-8) were considered for the arches. The compressive strength of masonry was assumed 2.4 MPa for KL1 and 3.2 MPa for KL3. The strength is further reduced by a confidence factor of 1.35 for KL1 and 1 for KL3.
Table 5-8 Knowledge levels according to Italian Code (NTC-08, 2009)

<table>
<thead>
<tr>
<th>Knowledge level</th>
<th>Description</th>
<th>Confidence factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>KL1</td>
<td>Geometrical evaluation, limited investigations on the construction details and limited investigations on the material properties</td>
<td>1.35</td>
</tr>
<tr>
<td>KL2</td>
<td>Geometrical evaluation, extensive investigations on the construction details and extensive investigations on the material properties</td>
<td>1.20</td>
</tr>
<tr>
<td>KL3</td>
<td>Geometrical evaluation, extensive and thorough investigations on the construction details and extensive and thorough investigations on the material properties</td>
<td>1.00</td>
</tr>
</tbody>
</table>

To evaluate the minimum and maximum stress levels acting on the main and secondary arches, Laterza et al. (2016) performed a structural analysis of the structure. For the minimum stress level only the self-weight was considered, while for the maximum stress level, the most unfavourable position of the load was considered. The stresses developed on the main and secondary arches are reported in Table 5-9 for the two knowledge levels.

Table 5-9 Stresses in the main and secondary arches according to Laterza et al., 2016

<table>
<thead>
<tr>
<th>Stress</th>
<th>Main arch</th>
<th>Secondary arch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$ (MPa)</td>
<td>KL1</td>
<td>KL3</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$ (MPa)</td>
<td>1.41</td>
<td>1.41</td>
</tr>
<tr>
<td>$\sigma_{\text{min}}$ (MPa)</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>$\Delta\sigma$ (MPa)</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>$S_{\text{max}}$ (%)</td>
<td>79.00%</td>
<td>43.90%</td>
</tr>
<tr>
<td>$S_{\text{min}}$ (%)</td>
<td>68.62%</td>
<td>38.13%</td>
</tr>
<tr>
<td>$\Delta S$ (%)</td>
<td>10.38%</td>
<td>5.77%</td>
</tr>
</tbody>
</table>
To apply the model proposed for the S-N-P curves (Equation 5-17), the 0.95 probability of survival SN curve (Equation 5-62 for L=0.95) is plotted in Figure 5-34 and Figure 5-35 for the main and secondary arches respectively for the two knowledge levels. The 0.95 probability of survival SN curve is considered as it represents an appropriate lower bound for conservative assessment. The maximum stress level on the arch and the loading cycles experienced to date are also depicted in the figures.

$$L = 10^{-0.1127(S_{\text{max}}\Delta S)^{3.9252}(\log N_f)^{3.8322}}$$  
Eq. 5-62

Due to small stress ranges $\Delta S$ and maximum stress levels on the arches (Table 5-9), the residual life under fatigue loading can be assumed infinite for the main arch for KL3 and for both KL1 and KL3 cases for the secondary arches. The number of loading cycles to failure $N_f$ is greater than $10^9$ giving a fatigue life of 2000 years. Normally modern bridges are designed for 120 years, however, masonry arch bridges last much longer e.g 300 years or more. For 300 years the expected loading cycles is $3 \times 10^8$. For the main arch for KL1 the number of loading cycles resulting from the SN curves is $3 \times 10^8$ or 300 years. Therefore the remaining service life is 220 years.
Figure 5-34 SN curve for 0.95 probability of survival, maximum stress level corresponding to main arches and number of sustained cycles to date for (a) KL1 and (b) KL3.
Figure 5: SN curve for 0.99 probability of survival, maximum stress level corresponding to secondary arches and number of sustained cycles to date for (a) KL1 and (b) KL2.
To plot the strain evolution curve with the loading cycles for the main arch for knowledge level KL1 ($S_{\text{max}}=79\%$) Equations 5-33, 5-37 and 5-43 were used for the different fatigue stages (Figure 5-36). For the Young’s modulus evolution curve, Equation 5-61 was employed (Figure 5-37). The sustained cycles to date over the fatigue life $N/N_f = 8 \times 10^7/3 \times 10^8 = 0.27$ ($N_f$ evaluated from SN curve) and also depicted in both plots.

The structure is experiencing stage II of the fatigue life. The strain at this stage is ca. 1.5 times the initial strain $\varepsilon_0$ and the Young’s modulus has decreased to ca. 96.5% of its the initial value $E_0$. At the end of the fatigue life Young’s modulus decreases to ca. 87.5% of $E_0$ for 79% maximum stress level. The Young’s modulus could be, therefore, reduced by a safety factor of 0.85 – 0.90 when assessing the structure under fatigue loading.

![Figure 5-36 Strain-loading cycles curve for 79% maximum stress](image)

178
5.6 Discussion

The purpose of this chapter was to develop mathematical expressions to characterise and reproduce various aspects of the behaviour of masonry under compressive fatigue loading. Similar expressions have been developed previously for masonry and concrete.

5.6.1 S-N curves for masonry

A mathematical model was proposed in Section 5.2.2 to predict the S-N-P curves for masonry. In order to identify the validity of the model for masonry under fatigue compression for wider application, fatigue data from other research studies are also analysed (Clark, 1994; Roberts et al., 2006; Tomor et al., 2013).
The fatigue test data by Clark (1994) is presented in Figure 5-38 together with the proposed S-N-P curves. Dry and wet masonry prisms were loaded up to 5 million cycles at 5 Hz frequency. The minimum stress level was 5% of the compressive strength. Prisms that did not fail were subsequently loaded under quasi-static compressive stress up to failure.

The proposed S-N-P model seems to provide a good estimate for dry masonry prisms. The model is, however, less accurate for saturated specimens, most of which fall below the 0.99 survival curve. Saturated specimens should be, therefore, analysed using separate equations. There is, however, insufficient test data for further analysis at the time of writing.

![Figure 5-38 Test data by Clark (1994) with proposed S-N-P curves (n=15)](image)

Roberts et al. (2006) performed a series of quasi-static and high cycle fatigue tests on three different types of masonry under dry, saturated and submerged masonry prisms. Although...
specimens were made with different mortar types and tested under different eccentricity ratios, they were analysed together, grouped based on the degree of saturation by Roberts et al. (2006). Additionally, Roberts et al. (2006) performed the tests under several different minimum stress levels ranging between 4-35% of the compressive strength. Based, however, on previous observation that the proposed model can not predict the fatigue life of saturated specimens, only results for dry specimens are presented in Figure 5-39 together with the proposed S-N-P curves. Test data are grouped according to minimum stress level and the 50% probability of survival is only presented to avoid entanglement.

The model does not seem to offer good approximation of the mean experimental data especially for low minimum stress levels. The test data are, however, limited to draw firm conclusions from. The disagreement of the proposed model and the test data of Roberts et al. (2006) could be attributed to several factors. Roberts et al. (2006) used stronger masonry types (from $f_c=5.83$ N/mm$^2$ to $f_c=11.85$ N/mm$^2$ for different prism types) compared to the current test data ($f_c=2.94$ N/mm$^2$). Roberts et al. (2006) also used three different prism types some of which included head joints and tested the prism under eccentric fatigue loading. Finally, the tests were performed under 5 Hz frequency. It is, therefore, important to study the effect of the compressive strength of masonry, eccentricity, frequency and prism type.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

The fatigue test data by Roberts et al. (2006) is presented in Figure 5-39 with proposed S-N-P curves for different minimum stress levels (n=13).

Figure 5-39 Dry test data by Roberts et al. (2006) with proposed S-N-P curves for different minimum stress levels (n=13)

The fatigue test data by Tomor et al. (2013) is presented in Figure 5-40 together with the proposed model for S-N-P curves. The minimum stress level was 10% of the quasi-static compressive strength and the maximum stress level ranged between 29% and 77% with limited available test data below 58%.

By excluding specimens that did not fail, the 0.50 probability of survival curve gives a good approximation of the mean fatigue life for masonry at each stress level. The 0.95 survival curve provides a satisfactory suitable lower bound and the 0.99 survival curve a more conservative lower bound.
Experimental and analytical investigations of brick masonry under compressive fatigue loading

Based on the previous examples, Equation 5.17 appears to provide an acceptable estimate for the fatigue life of centrally tested dry brick-masonry. The 0.50 probability of survival curve represents the mean fatigue life, the 0.95 survival curve an acceptable lower bound and the 0.99 survival curve a more conservative lower bound. The 0.05 survival curve represents an acceptable upper bound in most cases and the 0.1 survival curve a more conservative upper bound.

Test data presented by Clark (1994), Roberts et al. (2006) and Tomor et al. (2013) are rather limited and it has been observed based on the current experimental data that fatigue data scatter greatly. Tests were conducted under different ratios of minimum stress over compressive strength ($\sigma_{\text{min}}/f_c$). Clark (1994) used $\sigma_{\text{min}}/f_c$ of 5%, Roberts et al. (2006) between 4% and 35% and Tomor et al. (2013) 10%. This is another aspect that should be

Figure 5-40 Test data by Tomor et al. (2013) with proposed S-N-P curves (n=13)
further investigated. The S-N-P model seemed, however, to provide good prediction for different minimum stress levels.

The proposed S-N-P model is compared with models proposed by other researchers (Roberts et al., 2006; Casas, 2009; Tomor & Verstrynge, 2013). Figure 5-41 depicts the lower bound models and Figure 5-42 the models for mean values of fatigue data with the current test data. The comparison is carried out in two different figures as the prediction models by Roberts et al. (2006) and Tomor and Verstrynge (2013) do not have the ability to account for different confidence levels. Roberts et al. (2006) proposed a lower bound limit for the fatigue life of masonry, while Tomor and Verstrynge (2013) proposed an expression for the mean fatigue life. The model proposed by Casas (2009) is able to provide the S-N curves for masonry under fatigue for specified probabilities of survival, ranging between 0.50 and 0.95 and is depicted in both figures for different probabilities of failure.

The model from Equation 5.17 for 0.95 probability of survival, the model proposed by Casas (2009) also for 0.95 probability of survival and the model for the lower bound proposed by Roberts et al. (2006) are shown jointly in Figure 5-41. The model by Roberts et al. (2006) consists a lower bound of the test data but the configuration of the model does not follow the data points very closely. The model of Casas (2009) gives a better approximation of the test data in terms of curvature but does not provide a lower bound, especially for maximum stress levels between 60-80%. The proposed model in Equation 5.17 seems to provide a suitable lower limit following the test data closely and has the added benefit of allowing the probability of survival to be adjusted for a best fit.
The model from Equation 5.17 for 0.5 probability of survival from, the model proposed by Casas (2009) also for 0.5 probability of survival and model by Tomor & Verstrynge (2013) for the mean fatigue life are shown jointly in Figure 5-42. The model by Tomor & Verstrynge (2013) with correction factor $C=-1.5$ (identified to best fit current test data) seems to provide a good estimation of the mean test data but the configuration of the curve does not follow the data points very closely. This model also requires experimental data to identify parameter $C$ of Equation 5.4. The model by Casas (2009) is highly overestimating the fatigue life of masonry prisms at any stress level. Equation 5.17 due to its curvature appears to provide a suitable estimate for the mean test data.
5.6.2 Changes in strain (ε-N) during fatigue deterioration

In Section 5.3.2, a mathematical model was presented to characterise the change in strain (ε-N relationship) during fatigue loading. Three mathematical equations were proposed to characterise the ε-N behaviour of masonry during the three key stages of fatigue deterioration and the equation shows good agreement with the experimental data for the specific masonry types.

While Carpinteri et al. (2014) proposed a model only for stage II with fatigue experimental data needed to determine some of the parameters regarding the strain rate, the proposed new ε-N model for masonry predicts all three stages during fatigue deterioration. The proposed model also accounts for a longer duration of stage II of the fatigue life, from 10 - 90% of the total fatigue life of masonry instead of 10 - 80% as suggested by Carpinteri et al.
(2014). This difference is due to the scatter of the test data that indicated start of stage II between 6% and 14% and end of stage II between 76% and 94% of the fatigue life.

Holmen (1982) considered the effect of both minimum and maximum applied stresses in his $\varepsilon$-N model for concrete. The experimental tests within the current research were performed, however, under the same minimum induced stress and the proposed model does not, therefore, allow different minimum stress levels to be accounted for. More experimental data are required to further investigate effect of minimum stress level.

The comparison of the proposed model with the experimental data showed that the model can reliably predict the strain evolution for masonry (type B1M1) at any maximum stress level. No gaps exist at the intersection points between subsequent stages (accuracy of at least ten decimals) and the slopes of the curves coincide at these points, resulting to a differentiable function.

There are no other test data in the literature that can be used for further validation of the model. The validity of the proposed model is yet to be investigated for different masonry types and requires further test data. The effects of frequency, minimum induced stress, loading type and compressive strength also need to be investigated.

The model developed for the evaluation of the elastic and plastic strain evolution is based on the same concept as for the total longitudinal strain. The proposed model seems to provide a good approximation of the test data. Nevertheless, there is no relevant research on this topic and the results are based on limited experimental data. It cannot, therefore be proposed for wider application but can provide a starting point for future research.
5.6.3 Changes in the Young’s modulus (E) during fatigue deterioration

Finally, in Section 5.4.2 a mathematical expression for the evolution of Young’s modulus during fatigue deterioration was proposed. The E-N curve that was identified based on the experimental data does not exhibit the ‘S’ shape as seen for concrete (Holmen, 1982; Alliche, 2004; Breitenbucher & Ibuk, 2006; Zanuy, 2008) and no other relevant data is available for masonry. The stiffness deteriorates at a constant rate up to around 95% of the fatigue life and decreases suddenly just before failure. While the stiffness decreases up to ca. 40% of its initial value for concrete (Holmen, 1982), it only decreases up to ca. 25% for masonry. Instead of developing a three-part equation model for the change in stiffness, as proposed by Zanuy (2008), a single linear equation was developed to relate the normalised stiffness ratio (E/E₀) to the fatigue life. The effect of the minimum induced stress, the load frequency and the compressive strength were not considered.

5.7 Chapter summary

Analysis of the experimental data to obtain mathematical expressions for the prediction of the behaviour of masonry under long-term fatigue loading in compression has been presented in chapter 5. A new S-N-P model for masonry was developed and compared against available experimental data and analytical models. The proposed model was observed to provide a good approximation of the experimental data. A set of three equations was also proposed for the evolution of strain at different stages during fatigue deterioration. The percentages of strain corresponding to elastic and plastic components were also identified. A linear equation was proposed to predict the stiffness deterioration of masonry under fatigue. Finally, a practical example of application of the proposed
models to assess a masonry arch bridge under traffic loading was presented and the results were discussed and compared with data and relative models in literature.
6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

This chapter provides a review of the research findings and conclusions. How the research met the objectives through the findings will be justified and the limitations considered. The most important findings of the research are summarised and the associated implications on different professions discussed. Additionally, the contribution of this work to the body of knowledge in the field of masonry fatigue is justified and the conclusions reached through the experimental and the analytical studies are listed. Recommendations for future research on the fatigue deterioration of masonry are given at the end of the chapter.

6.2 Review of research objectives

In Chapter 1, the aim and specific objectives of the Thesis were stated. The aim was to identify the influence of compressive fatigue loading on the behaviour and mechanical properties of low-strength brick-masonry. This was addressed through six objectives.

**Objective 1: Evaluate current knowledge on the fatigue deterioration of masonry**

The first objective was to conduct a desktop survey to evaluate current knowledge in the field of fatigue deterioration of masonry. An extensive review of literature was conducted in Chapter 2. The historical background of research on fatigue was presented and the
processes taking place during fatigue were introduced for brittle materials, focusing mainly on concrete. Experimental test results and analytical models were reviewed in the literature for the S-N relationship for masonry and gaps in analytical understanding were identified in the response of masonry under fatigue loading.

**Objective 2: Generate experimental data on the response of masonry under quasi-static and fatigue compressive loading**

The methodology for collecting the experimental data was described in Chapter 3 and the results were presented in Chapter 4. Quasi-static compression tests were performed on B1 type clay-bricks and M1 mortar cubes to characterise the components of B1M1 type masonry. Based on the tests, the compressive strength and Young’s modulus were evaluated and the respective stress-strain curves plotted.

Quasi-static compressive tests were also performed on B1M1 masonry prisms (6 samples). Based on the test data the Young’s modulus and compressive strength were calculated. Stress-strain curves were plotted and the failure mechanism identified.

Long-term fatigue tests on masonry prisms under various maximum stress levels (55 - 80% of the compressive strength) were performed to study the fatigue deterioration of masonry (32 samples). The test data were used to plot the S-N (Stress-Number of cycles) and ε-N (Strain-Number of loading cycles) curves. The failure mechanism of masonry prisms under compressive cyclic loading was characterised and three distinctive stages of fatigue were identified. Pilot tests under compressive fatigue loading were specifically designed to study changes in stress strain curves during the fatigue life of masonry (32 samples). Curves were plotted and changes in configuration identified.
**Objective 3: Study the change in the stress-strain curves during fatigue deterioration and relate them to relevant studies for concrete**

A set of tests was designed to study changes in the stress-strain relationship during fatigue deterioration. Data collected by the LVDTs were used to plot the stress-strain curves at various stages during fatigue deterioration and for various maximum stress levels (chapter 4). Changes in the configuration of stress-strain curves were identified, correlated to the different stages of fatigue and compared to relative test results for unreinforced concrete (Crumley & Kennedy, 1977).

**Objective 4: Propose a mathematical model for S-N-P (Stress-Number of cycles-Probability of survival) relationships of low-strength masonry during compressive fatigue loading**

Data collected during the laboratory-based experimental tests were analysed in Section 5.2.2 to create a mathematical model for S-N-P (Stress-Number of cycles-Probability of survival) curves for low-strength brick-masonry under compressive fatigue loading. The S-N-P model was established by adapting an analytical expression proposed by McCall (1958) for unreinforced concrete. The model was modified to account for different minimum stress levels, validated against the test data and good agreement was observed. The proposed model was compared with S-N models published for masonry (Roberts et al., 2006; Casas, 2011; Tomor & Verstrynge, 2013) and was validated against published experimental data (Clark, 1994; Roberts et al., 2006; Tomor et al., 2013).
Objective 5: Propose a mathematical model for the evolution of strain for low-strength masonry during compressive fatigue loading

Experimental data were processed in Chapter 4 and ε-N (Strain-Number of cycles) curves were plotted for all tested stress levels. The configuration of ε-N curves indicated that changes in strain during fatigue deterioration can be divided into three distinct stages. In stage I initiation of micro-cracks takes place during the first 10% of the fatigue life. In stage II cracks grow steadily up to 90% of the fatigue life and in stage III coalition of micro-cracks into macro-cracks takes place, leading to failure.

Based on the principles used by Holmen (1982) and Zanuy (2008) for concrete, a new model was proposed for the evolution of strain for masonry under fatigue loading. The proposed model consists of a separate equation for each of the three stages during fatigue deterioration. The model was validated against experimental data and good agreement was observed. A similar model was also developed for predicting the evolution of elastic and plastic strain during fatigue for specific maximum stress levels.

Objective 6: Propose a mathematical model for the evolution of Young’s modulus for low-strength masonry during compressive fatigue loading.

Experimental data were post-processed to investigate the evolution of Young’s modulus during fatigue deterioration in Section 5.4.2. The configuration of the E-N (Young’s Modulus-Number of cycles) curve at different stress levels was studied and trends identified. The Young’s modulus was found to decrease at a constant rate up to ca. 95% of the fatigue life and different stages were not clearly observed. A prediction model based
on the hypothesis of constant decrease of the Young’s modulus with increased loading cycles was proposed.

### 6.3 Research limitations

A number of limitations need to be considered for this work. As in most studies involving experimental tests, the majority of the limitations were introduced during the research design stage.

The research is focusing on the behaviour of low-strength brick masonry under compressive fatigue loading. The issue is most relevant to masonry arch bridges in the traffic network that experience dead and live load and are often subjected to great long-term stress variations. The type of masonry in bridges varies widely depending on the age of construction, location, choice of materials and construction form. As it would be unfeasible to investigate the behaviour of a wide range of masonry types, a worst-case scenario was considered. A low-strength type of masonry was selected that is most relevant to masonry arch bridges built for the British canals typically in the 18th century with low-strength hand-made clay bricks and lime mortar (denoted as B1M1 type masonry in the research). Test results and analytical models are, therefore, most relevant to low-strength masonry with lime mortar. Different types of masonry should be tested before the models can be applied for a wider range of masonry.

In terms of capacity, masonry can suffer deterioration due to centric or eccentric compressive, shear or tensile loading. Deterioration can be caused by static, quasi-static, cyclic or long-term fatigue loading. The current research investigates the combination of quasi-static and fatigue loading defined by a minimum and maximum stress level. Naturally,
the level or minimum and maximum stress levels vary from structure to structure. Due to confines of time, the research is focusing on some worst-case scenarios with minimum stress around 10% of the compressive strength and a selection of maximum stress levels (between 55 - 80% of the compressive strength). Higher minimum stress levels can have additional impact on the test results and should be investigated. The frequency of fatigue loading was set to 2Hz to represent the flow of traffic at ca. 40-50 Km/hour speed over short-span masonry bridges. The effect of loading frequency on the test results should also be investigated for wider application.

In terms of the duration of the tests, fatigue testing of specimens can last from a few minutes to several months. The lower the (maximum) stress, generally the longer it took for the specimens to fail. Some of the prisms were loaded up to $10^7$ cycles (55% maximum stress) that took about two months of continuous testing and indicated the practical limit for the physical testing. It was, therefore, not practical to collect test data for stress levels below 55% maximum stress. Although it may not be critical for masonry arch bridges in the waterways network, the proposed models are yet to be validated for very low stress levels.

Several limitations apply to the mathematical models as well. The mathematical expression developed for the S-N-P curves for masonry does not apply for saturated specimens or for tests under eccentric loading. The proposed models for the evolution of strain and Young’s modulus during fatigue life takes into account only the maximum applied stress. The impact of the stress range and the minimum stress is, however, undeniable and the model should be validated against test data for various minimum stresses and updated. Additionally, the model described for the prediction of the elastic and plastic strains was validated against
test results for 63% and 68% maximum stress levels and cannot be proposed for wider use. It constitutes, however, a starting point for future research.

6.4 Review of research findings

Long-term fatigue of masonry constitutes progressive deterioration in the behaviour and mechanical properties of masonry at stress levels lower than the compressive strength. Despite different stress levels, the failure mechanisms of prisms under compressive fatigue loading were very similar to those under compressive quasi-static loading. All prisms failed by developing vertical cracks at the sides along the headers (narrow sides).

Experimental data confirmed the findings of other researchers (Clark, 1994; Roberts et al., 2006; Tomor et al., 2013) that, in general, the number of loading cycles to failure decreases with increased stress levels. The variability of the test data is, however, quite large and probabilistic methods are required to define the relationship between stress and loading cycles.

The stress-strain curves, plotted at various stages during the fatigue life of specimens, indicate changes in the material properties during fatigue loading. As the curve shifts to larger strains with increased cycles of loading, due to accumulation of residual strains, the shape of the graph is altered. Three distinct stages were observed in the evolution of stress-strain curves during fatigue deterioration. The residual strains grow rapidly during the first few cycles of loading, the rate of increase in strain becomes smaller and almost constant in the second stage and strain rises rapidly just before failure. Changes in the curve configurations were also identified. From being initially linear or concave, before the application of cyclic load, the curve becomes convex with respect to the strain axis and the
Curvature increases with increased loading cycles. Strains at failure were notably higher when specimens were subjected to cyclic loading. No relevant experimental test data has been published in the literature for masonry, but findings are similar to concrete (Crumley & Kennedy, 1977; Holmen, 1982).

Experimental data were also used to plot the maximum and minimum total longitudinal strains against the number of cycles at different maximum stress levels. The evolution of strain with the loading cycles appears to exhibit an S-shape with three distinct stages, similarly to the stress-strain curves. The first stage is up to ca. 10% of the fatigue life and is characterised by rapid growth of strain due to initiation of micro-cracks. The second stage, representing the largest part of the curve (ca. 10 - 90% of fatigue life) is characterised by gradual increase of the strain at a constant rate due to growth of micro-cracks. In the final stage, strains grow rapidly until ultimate failure occurs due to the coalition of micro-cracks into macro-cracks. The slope of the curve in the second stage becomes steeper for specimens that fail earlier at each maximum stress level. Total recorded strain at failure is larger in tests that last longer due to the increased effect of creep.

An S-N-P (Stress-Number of cycles-Probability) relationship was proposed for brick masonry for long-term compressive fatigue loading (Equation 6-1). The proposed model was based on a similar model used for concrete (McCall, 1958) and validated against test results undertaken in the current research and test results published in the literature (Clark, 1994; Roberts et al., 2006; Tomor et al., 2013). The model was, subsequently, compared with other S-N relationships (Roberts et al., 2006; Casas, 2011; Tomor & Verstrynge, 2013) and proved to be reliable in predicting the fatigue life of dry masonry at any required probability level.
Based on the experimentally obtained $\varepsilon$-N (strain-loading cycles) curves and models published for concrete (Holmen, 1982; Zanuy, 2008), a prediction model was developed for masonry to allow the normalised strain ratio ($\varepsilon/\varepsilon_0$) to be calculated at any stage during the fatigue life of the specimens. The model consists of three distinct branches representing the three stages of fatigue (Equations 6-2 to 6-4). The first branch occupies 10% of the fatigue life and is characterised by a second order parabolic relation. The second branch is expressed by a linear relationship between 10 - 90% of the fatigue life. The third branch of the curve is also expressed by a second order parabolic equation and represents the last 10% of the fatigue life. Continuity of the curve in terms of mathematical values and curvature was achieved at the intersection points between subsequent stages. The prediction model was validated against the experimental data and good agreement was found.

For $\frac{N}{N_f} \leq 0.1$

$$\frac{\varepsilon_{max}}{\varepsilon_0} = \left(547.9S_{max}^2 - 635.8S_{max} + 147.12\right) \left(\frac{N}{N_f}\right)^2 - \left(97.35S_{max}^2 - 111.58S_{max} + 23.343\right) \frac{N}{N_f} + 1$$ Eq. 6-2

For $0.1 < \frac{N}{N_f} \leq 0.9$

$$\frac{\varepsilon_{max}}{\varepsilon_0} = \left[12.23S_{max}^2 - 15.58S_{max} + 6.081\right] \frac{N}{N_f} - 5.479S_{max}^2 + 6.358S_{max} - 0.4712$$ Eq. 6-3
For \( \frac{N}{N_f} \leq 1 \)

\[
\frac{\varepsilon_{\text{max}}}{\varepsilon_0} = [781.9S_{\text{max}}^2 - 1488.8S_{\text{max}} + 727.02]\left(\frac{N}{N_f}\right)^2
\]

\[
+ [-1395.19S_{\text{max}}^2 + 2664.26S_{\text{max}} - 1302.555]\frac{N}{N_f}
\]

\[
+ [627.86S_{\text{max}}^2 - 1199.57S_{\text{max}} + 588.415]
\]

The experimental data from pilot tests used to develop the stress-strain curves were further processed to evaluate the growth of plastic and elastic strains during fatigue deterioration. Plastic strain was found to follow the same evolution as total strain, while the elastic one exhibits a nearly linear growth. A prediction model was generated for the proportion of plastic over total strains (\(\varepsilon_\text{pl}/\varepsilon_\text{tot}\)) following the procedure described for the total strains. Despite the fact that the model shows strong correlation with the experimental data, it cannot be proposed as a general model as it is based only on limited number of test data (63% and 68% maximum stress levels).

Finally, evolution of the Young’s modulus was studied during fatigue deterioration. While for concrete, the evolution of the Young’s modulus displays an S-shape configuration (Crumley & Kennedy, 1977; Holmen, 1982; Zanuy, 2008), for masonry it decreases steadily up to ca. 95% of the fatigue life and decreases rapidly during the last few loading cycles. The change in behaviour during the last cycles is not apparent for all prisms. The maximum decrease in Young’s modulus is up to around 25% of its initial value. As the decrease in Young’s modulus is stable during the majority of the fatigue life, a linear equation was proposed (Equation 6-5).

\[
\frac{E}{E_0} = 1 - (3.0181S_{\text{max}}^3 - 5.6894S_{\text{max}}^2 + 3.5118S_{\text{max}} - 0.6175)\left(\frac{N}{N_f}\right) \quad \text{Eq. 6-5}
\]
6.5 Application of the research findings

As the majority of masonry arch bridges are over 100 years old and subjected to constantly increasing traffic weight, speed and density, the effect of fatigue loading on the behaviour and the mechanical properties of masonry is becoming an increasingly relevant issue. The outcomes of the current research may be beneficial for various professionals working with existing bridges, as well as those working with historical heritage.

For bridge management, information on the rate of deterioration and remaining service life is essential to optimise assessment and inspection techniques and minimise the cost of maintenance. S-N-P curves can provide a useful tool to help evaluate the rate of deterioration and remaining service life of masonry arch bridges at different confidence levels, based on material properties and traffic load levels. Optimising the weight, speed and frequency of traffic could also help reduce deterioration, particularly in older and weaker bridges.

For structural engineers the process of progressive, irreversible damage in a material under cyclic loading is of great importance. The structural changes are associated with progressive growth of internal micro-cracks, which leads to significant growth of plastic strain. At a macro-level this process leads to changes in the mechanical properties of the material (Lee & Barr, 2004). Therefore, a time-dependant model able to predict the mechanical changes of masonry with the number of cycles is necessary to study the influence of fatigue on the structural behaviour of masonry. Changes to the rate of growth of deformation during long term monitoring of masonry arch bridges under traffic can be associated with different stages of fatigue. The proposed prediction model for the law of evolution for the total
longitudinal strain with the number of cycles could be adopted to evaluate the remaining service life.

Findings on the stiffness deterioration of masonry can be used to propose a reduction factor of the Young’s modulus of masonry during the design or assessment of masonry structures to account for the influence of fatigue. The reduction factor should range from 0.9 to 0.75 of the initial value of the Young’s modulus, depending on the stress level. Finally, the evolution laws of the mechanical properties of masonry subjected to repeated loading can be adapted by commercial finite element software packages to develop time-dependant models for the analysis of masonry under fatigue.

Fatigue test data and the proposed mathematical model for the SN curves can also be fed into the SMART method (Melbourne et al., 2007). The SMART method can, therefore, be used to quantify the residual life of brick masonry arch bridges, especially canal bridges made of masonry with similar characteristics (compressive strength and Young’s modulus) as the one used for current research. The model should be considered for failure modes associated with compressive loading (crushing).

6.6 Contribution to the body of knowledge

The available experimental data prior to this research were limited (Clark, 1994; Ronca et al., 2004; Roberts et al., 2006; Tomor et al., 2013) and the tests were performed under various frequencies, minimum stresses, eccentricity ratios, degrees of saturation, masonry types and prism types. The small number of samples and the variability of influencing factors within each study would complicate comprehension of the fatigue behaviour of masonry and the effect of parameters. Experimental tests conducted within this study
provide an important database on the effect of compressive fatigue loading on the behaviour of low-strength brick masonry. A total of 64 B1M1 masonry prisms were tested under the same minimum stress level and frequency. Information on the fatigue life, the evolution of strain, duration of different stages of fatigue and failure mechanisms were collected. Stress-strain curves at different stages of the fatigue life were presented for the first time through specifically designed tests. Results revealed changes in the configuration of the curves with increased loading cycles.

A mathematical model for the S-N-P (stress-number of cycles-probability) curves was proposed based on the experimental data. This model provides the possibility to predict the SN curves of a specific masonry type at any desired probability of failure and can be used, therefore, to define curves for the mean, upper and lower limits of fatigue life. Previously proposed models would allow the user to plot the SN curves at predefined probabilities of failure and not for the whole range of probabilities.

Limited $\varepsilon$-N (strain-number of cycles) curves for masonry are available in the literature (Abrams et al., 1985; Carpinteri et al., 2014) and mainly for low-cycle fatigue. The $\varepsilon$-N curves were plotted throughout the fatigue life of masonry prisms and three distinct stages were identified. A set of three equations were proposed for the first time to predict the evolution of strains at each stage of the fatigue life of masonry.

Deterioration of the Young’s modulus of masonry under fatigue loading was also studied for the first time. The E-N (Young’s modulus-number of cycles) curves were plotted, a mathematical expression to predict the deteriorated stiffness was established, and the maximum decrease of the Young’s modulus identified.
6.7 Conclusions

Quasi-static and long-term fatigue tests were performed on masonry prisms constructed of low-strength bricks to study the changes in material properties during fatigue deterioration of low-strength brick masonry. The main conclusions from the experimental tests can be summarised as follows.

i. The failure mechanism under compressive fatigue loading is similar to compressive quasi-static loading. Vertical cracks develop through the mortar joints and bricks across the narrow sides of the prisms, leading to failure.

ii. The $\varepsilon$-N (Strain-Number of cycles) curve recorded under fatigue loading exhibits a typical ‘S’ configuration, similar to concrete.

iii. Three distinct stages were identified during fatigue deterioration in the evolution of strain. In stage I (0-10% of fatigue life), the strain grows rapidly due to initiation of micro-cracks. Stage II (10-90% of the fatigue life) occupies the majority of the fatigue life during which the strain grows steadily. In stage III (90-100% of the fatigue life) the strain grows rapidly again, due to the coalition of micro-cracks to macro-cracks, resulting in failure.

iv. Configuration of the stress-strain curve changes with the number of loading cycles from concave, with respect to the strain axis, to convex with greater curvature for increased loading cycles.

v. The total recorded strain under quasi-static loading is notably lower than for fatigue loading. Strain increases as the maximum applied stress decreases.
The collected experimental test data were analysed and mathematical models were proposed to relate stress, strain and Young’s modulus to the loading cycles.

i. The proposed stress - number of cycles - probability of survival (S-N-P) model shows good agreement with the test results and reliably predicts the fatigue life of dry low-strength B1M1 masonry at any desired confidence level. More specifically, the 0.50 probability of survival curve represents a good approximation of the mean fatigue life, while the 0.95 probability of survival curve represents an appropriate lower bound.

ii. The proposed ε-N model provides an acceptable prediction for the mean test data for low-strength B1M1 masonry at any maximum stress level. The initial assumptions that a parabolic equation can describe stage I and III, while a linear equation is suitable for stage II proved to provide a good approach of the configuration of the curve, as revealed by the experimental data.

iii. The percentages of the elastic and plastic strain were studied and changes in the two components during fatigue identified. A set of three equations was proposed to predict the percentage of the elastic and plastic strain during each stage of fatigue (second order parabolic for stage I and III and linear for stage II). At failure the percentage of plastic strain is greater for lower stress levels (ca. 80% for 63% maximum stress and ca. 50% for 68% maximum stress).

iv. The Young’s modulus deteriorated at a constant rate up to about 95% of the fatigue life and decreases suddenly just before failure. The residual Young’s modulus just before failure ranges between 97.5 – 75% of its initial value (E₀).

v. A linear equation can adequately predict the decrease of the Young’s modulus during the fatigue life of masonry.
6.8 Recommendations for future research

An experimental study of fatigue is a time-consuming process and only allows a limited number of issues to be investigated within certain time limits. To gain a better understanding of the influencing factors for fatigue deterioration, a series of further parameters would need to be investigated. It should, however, be borne in mind that the research is aiming to provide useful data and prediction models for the assessment of brick masonry, particularly for historical masonry arch bridges, for which not all factors are of particular significance.

- A range of further brick masonry types should be tested, that are relevant to masonry arch bridges. In the UK the main masonry types are associated with waterways, railways and recent bridges as discussed in Chapter 3.

- Although the test specimens were built without headers (representing multi-ring arches without headers), the influence of head joints and the thickness of mortar joints should also be investigated.

- Historical masonry is often incorrectly repointed. The use of stiff cement mortar can cause serious deterioration within only a few years. The effect of mortar type on the capacity of masonry under quasi-static and fatigue loading are serious concerns and would be of great importance to bridge managers.

- During the tests, compressive loading was applied centrally on prisms. In masonry arches loading is, however, mostly eccentric that is likely to increase the rate of deterioration during fatigue loading notably. Eccentric fatigue loading needs to be further investigated.
The minimum stress level was constant during the tests (10% of the compressive strength) to investigate the worst-case scenario. The effect of further minimum stress levels that are relevant to the dead load in masonry arch bridges should also be investigated.

The maximum stress level for fatigue testing of masonry prisms within the research was between 55 - 80% of the compressive strength. While metals have fatigue limits around 50% of the ultimate strength, no such limit has been identified for masonry. If time constraints could be overcome, it would be useful to test masonry samples below 50% of the compressive strength. The number of fatigue cycles and maximum stress levels should be related, however, to the likely number of cycles relevant to masonry arch bridges during their life expectancy and relevant live loads.

The effect of variable amplitude fatigue loading (changing minimum and maximum stress levels and sequence of stress ranges) should be investigated with respect to Miner’s rule.

The effect of loading frequency would need to be investigated to identify possible influence on the fatigue deterioration.

Test results would need to be related to representative masonry arch bridge types, such as minimum and maximum stress levels related to dead and live loads, typical working stress levels in various parts of masonry bridges, expected service life and changes in traffic loading. Testing medium and large-scale masonry arch bridges would provide useful information on the scale-effects.

The analytical expressions proposed for the evolution of strain and Young’s modulus should be updated based on new experimental data.
REFERENCES


Experimental and analytical investigations of brick masonry under compressive fatigue loading


